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A Tale of Two Networks: Common Ownership and Product Market Rivalry

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A Tale of Two Networks: Common Ownership and Product Market Rivalry*

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Abstract

We study the welfare implications of the rise of common ownership in the United States from 1994 to 2018. We build a general equilibrium model with a hedonic demand system in which firms compete in a network game of oligopoly. Firms are connected through two large networks: the first reflects ownership overlap, the second product market rivalry. In our model, common ownership of competing firms induces unilateral incentives to soften competition. The magnitude of the common ownership effect depends on how much the two networks overlap. We estimate our model for the universe of U.S. public corporations using a combination of firm financials, investor holdings, and text-based product similarity data. We perform counterfactual calculations to evaluate how the efficiency and the distributional impact of common ownership have evolved over time. According to our baseline estimates the welfare cost of common ownership, measured as the ratio of deadweight loss to total surplus, has increased nearly tenfold (from 0.3% to over 4%) between 1994 and 2018. Under alternative assumptions about governance, the deadweight loss ranges between 1.9% and 4.4% of total surplus in 2018. The rise of common ownership has also resulted in a significant reallocation of surplus from consumers to producers.

JEL Codes: D43, D85, E23, L16, G23, G34

Keywords: common ownership, concentration, networks, institutional investors, oligopoly

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1 Introduction

The U.S. economy has experienced two significant trends in concentration over the last three decades. First, revenue and employment concentration have increased considerably across a broad range of industries (Grullon, Larkin and Michaely, 2019). Second, the ownership of corporate equity has become concentrated in the hands of a few large institutional investors (Ben-David, Franzoni, Moussawi and Sedunov, 2020). This latter trend is also known as the rise of common ownership (Azar, 2012; Gilje, Gormley and Levit, 2020; Backus, Conlon and Sinkinson, 2021b). This paper studies the increasing concentration of the overlapping networks of product market competition and firm ownership and shows that the resulting welfare costs and distributional consequences are significant.

Common ownership refers to an ownership arrangement under which large investors own shares in several competing firms. The tremendous increase of common ownership has raised concerns among policymakers (see, for example, Phillips (2018) and Vestager (2018)) because it may lessen firms' economic incentives to aggressively compete against each other. If firms make strategic decisions to maximize the profits accruing to their investors, common ownership can lead firms to (partially) internalize the effect of aggressive market decisions on their competitors' profits. This may induce firms to produce lower quantities or to charge higher prices, ultimately leading to deadweight and consumer surplus losses.

This mechanism of harm of common ownership is supported by a long-standing and growing academic literature, starting with Rubinstein and Yaari (1983) and Rotemberg (1984), that studies oligopolistic behavior in the presence of common ownership. In response to the secular rise of common ownership and the concurrent surge of empirical research on its anticompetitive effects (see Schmalz (2018) for a recent survey and Shekita (2021) for a collection of specific examples), antitrust authorities and financial regulators around the world (including the Department of Justice, the Federal Trade Commission, the European Commission, and the Securities and Exchange Commission) have begun studying policy measures to address them.[1] Despite the enormous academic and policy interest in common ownership, as of today, there has been no attempt to quantify its aggregate welfare impact.

In this paper, we analyze the economy-wide welfare cost and distributional effects of common ownership from a theoretical and empirical perspective. First, we develop a general equilibrium model in which granular firms compete in a network game of oligopoly. Building on the rich

[1] Solomon (2016) reported on an investigation based on Senate testimony by the head of the Antitrust Division, the Federal Trade Commission (2018) featured a hearing on common ownership, and Vestager (2018) disclosed that the European Competition Commission is "looking carefully" at common ownership given indications of its increase and potential for anticompetitive effects. The Federal Trade Commission and Department of Justice (2022) are currently requesting public comments on how the agencies can modernize enforcement of the antitrust laws regarding mergers including the merger "guidelines' approach to common ownership and horizontal stockholding." For other recent activity, see OECD (2017), European Competition Commission (2017), and Jackson (2018).

literature on linear-quadratic network games (Ballester, Calvó-Armengol and Zenou, 2006; Ushchev and Zenou, 2018; Galeotti, Golub and Goyal, 2020), the firms in our model are connected through two large networks: the first reflects ownership overlap, the second product similarity.

Second, we estimate the model using data on firm financials, text-based product similarity (Hoberg and Phillips, 2016), and institutional investor holdings (Backus et al., 2021b) covering the universe of U.S. publicly listed corporations from 1994 to 2018. We perform counterfactual calculations to evaluate how the efficiency and the distributional impact of common ownership have evolved over this period, finding large negative consumer welfare effects.

Our model has two distinctive features. First, following the literature on hedonic demand (Lancaster, 1966; Rosen, 1974) it leverages the Generalized Hedonic-Linear (GHL) demand system recently developed by Pellegrino (2019). This demand system is based on the assumption that there is a representative consumer who has quadratic preferences over product characteristics (as opposed to products). The cross-price elasticity of demand between any two products is thus proportional to a metric of product similarity which captures whether two products contain similar attributes. This setup allows us to estimate, using the dataset of Hoberg and Phillips (2016), realistic cross-price demand elasticities specific to each firm pair and year, without having to take a stance on industry boundaries. Second, firms make strategic supply decisions with the objective of maximizing a weighted sum of profits earned by their investors, with each investor receiving a weight proportional to its ownership stake (Azar, 2012; López and Vives, 2019; Backus et al., 2021b; Azar and Vives, 2021a). This setup is isomorphic to each firm maximizing a weighted sum of its own profits and its competitors' profits, with each other company receiving a weight proportional to a well-defined measure of common ownership that can be computed using 13(f) institutional investor holdings data. A key feature of our model is that the anticompetitive effects of common ownership depend on the overlap between the two networks of product similarity and common ownership.

Although the increase in common ownership is already well documented and a number of empirical papers have provided evidence for anticompetitive effects of common ownership on prices, quantities, markups, managerial incentives, and profitability, no paper has estimated the economy-wide welfare cost of common ownership. Taking as given that common ownership *does* affect competitive behavior, how large are the resulting product market welfare costs of the increase in common ownership and industry concentration in the U.S. economy over the past two decades?[2] Answering this question requires a model that is both tractable and flexible enough to accommo-

[2]Our model and empirical analysis abstract away any labor market effects of common ownership which may result from enhanced employer power as in the theoretical analysis of Azar and Vives (2021a). We further consider neither coordinated anticompetitive effects of common ownership which may result from explicit or tacit collusion between firms or owners as documented by Shekita (2021) nor beneficial effects such as the internalization of innovation spillovers (Antón, Ederer, Giné and Schmalz, 2018; Gibbon and Schain, 2020). We also do not analyze the effect of common ownership on risk-taking and portfolio diversification by financial institutions, which has been studied separately by Galeotti and Ghiglino (2021). Thus, our analysis focuses exclusively on the welfare *costs* of *unilateral product market* effects of common ownership.

date the complex overlapping networks of product market competition and ownership that exist among public firms. The principal contribution of our paper is to propose such a model and to practically estimate it with data on product similarity and ownership networks.

We first visualize the two networks of product similarity and common ownership in which firms are embedded. The network of product similarities displays a pronounced community structure. Large groups of firms tend to cluster in certain areas of the network. In contrast, the network of common ownership has a hub-and-spoke structure with a large proportion of firms sharing significant overlap and the remainder of largely unconnected firms at the periphery. Across the distribution of firm pairs there is little correlation between product similarity and common ownership.

Next, we take the model to the data. Our model estimation reveals three broad patterns. First, the welfare costs of common ownership are significant, but not as large as the welfare costs of oligopoly. We estimate that in 2018, the most recent year of our sample, the deadweight loss of oligopoly (as measured by the loss in total surplus due to firms competing à la Cournot as opposed to competing as under perfect competition) amounts to about 11.5% of total surplus. Common ownership leads to an additional deadweight loss of 4% of total surplus (as measured by the additional loss in total surplus due to firms internalizing overlap in ownership with their competitors when competing à la Cournot). Second, the welfare losses of common ownership fall entirely on consumers. We estimate that in 2018 common ownership raises aggregate profits by \$378 billion (from \$5.261 trillion to \$5.639 trillion), but lowers consumer surplus by \$799 billion (from \$4.113 trillion to \$3.314 trillion). Third, the negative effects of common ownership on total welfare and consumer surplus have grown considerably over the last two decades. Whereas common ownership reduced total surplus by a mere 0.3% in 1994 this deadweight loss increased more than tenfold to 4% in 2018. Over the same time period, common ownership raised corporate profits by 1% in 1994 and 6.6% in 2018, but lowered consumer surplus by less than 2% in 1994 and almost 20% in 2018.

We further explore how alternative assumptions about corporate governance modify our results. Rather than investors influencing firm decisions exactly in proportion to their ownership stakes, larger investors may exert influence that exceeds the size of their stake.[3] Under such an alternative "superproportional influence" assumption, common ownership has essentially identical effects on deadweight loss, corporate profits, and consumer surplus. When we assume that only blockholders (i.e., shareholders holding 5% or more of a company's stock) can exert influence or that large diversified owners have limited attention we find that common ownership still leads to a deadweight

[3]Several large institutional investors such as BlackRock and TIAA-CREF have also argued for stronger "stakeholder capitalism" which takes the interests of stakeholders other than owners (e.g., employees and consumers) into account and seeks to internalize all the externalities that companies impose on "the society where they work and operate" (Fink, 2020), as well as to support broader goals of social responsibility (Hart and Zingales, 2017; Oehmke and Opp, 2019; Broccardo, Hart and Zingales, 2020). Our analysis builds on the arguably less ambitious assumption that investors influence companies to partially internalize only the effect on product market profits that their corporate conduct imposes on other firms in the same investors' portfolio.

loss of 2.5% of total surplus, raises firm profits by almost 5%, and lowers consumer surplus by almost 13% of total surplus in 2018. Finally, we show that our conclusions about the significant welfare and distributional consequences of common ownership are also robust to different assumptions about firm fixed costs and the measurement of intangible capital.

Our paper contributes to several literatures. First, this paper builds on the macroeconomic networks literature (Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi, 2012; Carvalho, 2014; Acemoglu, Ozdaglar and Tahbaz-Salehi, 2017; Carvalho and Tahbaz-Salehi, 2019; Baqaee and Farhi, 2020b; Liu and Tsyvinski, 2020; Carvalho, Nirei, Saito and Tahbaz-Salehi, 2021a; Carvalho, Elliott and Spray, 2021b). Whereas those papers focus on input-output networks we study networks of ownership and product market rivalry. As a result, our work shares similarities with Bloom, Schankerman and Van Reenen (2013) who empirically study innovation spillovers through product market and technology networks, and with Chen, Zenou and Zhou (2018, 2021) and Galeotti, Golub, Goyal, Talamàs and Tamuz (2021) who theoretically analyze the role of market structure and market power in networks.

Second, our work is related to the growing body of academic work on markups and industry concentration (De Loecker, Eeckhout and Unger, 2020; De Loecker and Eeckhout, 2018; Pellegrino, 2019; Syverson, 2019; Autor, Dorn, Katz, Patterson and Van Reenen, 2020; Döpper, MacKay, Miller and Stiebale, 2021).[4] By incorporating hedonic demand as well as data on product market similarity and ownership we characterize competitive interactions between firms that differ by their productivity as well by their products' characteristics and their ownership, including the connections that exist between the latter two features.

Finally, our paper is related to the burgeoning literature on common ownership. Several empirical papers (Matvos and Ostrovsky, 2008; Azar, Schmalz and Tecu, 2018; Boller and Scott Morton, 2020; Newham, Seldeslachts and Banal-Estanol, 2019; Xie and Gerakos, 2020; Li, Liu and Taylor, 2020; Antón, Ederer, Giné and Schmalz, 2020; Dennis, Gerardi and Schenone, 2020; Eldar, Grennan and Waldock, 2020; Lewellen and Lowry, 2021; Azar, Raina and Schmalz, 2021; Backus, Conlon and Sinkinson, 2021a; Saidi and Streitz, 2021) investigate whether common ownership affects firm decisions and industry outcomes (e.g., prices, quantities, markups, entry, managerial compensation, innovation). The evidence is mixed and growing as this is a very active area of research. Results vary across industries, outcome variables, and the specific methodologies used to estimate the various effects of common ownership.

Azar and Vives (2021a) theoretically study common ownership in a general equilibrium setting in which symmetric common ownership across identical, symmetrically-differentiated oligopolistic firms leads to additional market power with respect to both product market competition and labor hiring. In contrast to their paper, our theoretical model assumes competitive labor markets to focus

[4] To our knowledge the earliest empirical study estimating the deadweight loss resulting from firms' product market power in the U.S. economy is Harberger (1954). Other seminal contributions to this literature include Kamerschen (1966), Bergson (1973), and Cowling and Mueller (1978).

on the product market effects of common ownership but allows for arbitrary size differences and substitution patterns between firms. Furthermore, in addition to laying out a theoretical framework, we structurally estimate our model using granular, time-varying firm-by-firm and firm-by-investor network microdata and obtain detailed welfare estimates, including the effect of common ownership on individual firms' profits.

In sum, a key difference between our study and all previous contributions is that we espouse a macro-structural methodological framework that combines theory and data with the objective of answering a new and entirely distinct research question: Assuming that firms *do* maximize shareholder value (as opposed to own-firm profits), what are the economy-wide welfare and distributional consequences of common ownership?

By taking a network approach to modelling inter- and intra-industry competition, as predicated by Elliott and Galeotti (2019), we can overcome the problem of external validity (a natural limitation of industry studies), as we compute the welfare impact of common ownership for a broad set of industries. Additionally, because we model the product market as a network, we do not face the problem of having to arbitrarily define the relevant product markets, which is a source of model uncertainty in industry studies. Our analysis shows that, at the aggregate level, common ownership can generate significant distortions and reallocation of surplus—even under conservative assumptions about the effect of common ownership on corporate governance.

The remainder of the paper proceeds as follows. Section 2 develops our theoretical model. Section 3 describes the data and Section 4 reports the empirical results for the baseline model of corporate governance. Section 5 provides additional empirical results under alternative corporate governance and cost structure assumptions. Section 6 concludes.

2 Theoretical Model

We develop a general equilibrium model in which granular firms with overlapping ownership compete in a network game of Cournot oligopoly. Firms are connected through two large networks: the first reflects ownership overlap, the second product similarity. We characterize the equilibrium of this economy and then compare it to several counterfactual scenarios in which firms maximize objective functions that are different from those of their investors.

2.1 Generalized Hedonic-Linear (GHL) Demand System

There is a representative agent who is a consumer, worker, and owner. This representative agent consumes all the goods produced in the economy, supplies labor as a production input, and receives income from owning shares of all the firms in the economy.

Our economy has n firms, indexed by $i \in \{1, 2, \ldots, n\}$. Each firm produces a single differentiated product such that there are n products in the economy. The representative agent has hedonic demand (Lancaster, 1966; Rosen, 1974) and thus values each product as a bundle of its $m + n$

constituent characteristics.[5]

Each product has two types of characteristics: m common characteristics indexed by $\iota \in \{1, 2, \ldots, m\}$ and n idiosyncratic characteristics. Because the idiosyncratic characteristics are specific to product i and are not present in other products, they have the same index i. The m-dimensional column vector $\mathbf{a}_i$ describes product i where the scalar $a_{\iota i}$ is the number of units of common characteristic ι of product i. Without loss of generality we assume that the column vector $\mathbf{a}_i$ is of unit length:

$$\mathbf{a}_i \;=\; \begin{bmatrix} a_{1i} & a_{2i} & \ldots & a_{mi} \end{bmatrix}' \tag{2.1}$$

$$\text{such that} \qquad \sum_{j=1}^{m} a_{ji}^2 = 1 \quad \forall\, i \in \{1, 2, \ldots, n\} \tag{2.2}$$

We combine the product-specific vectors $\mathbf{a}_i$ of common characteristics in the $m \times n$ matrix $\mathbf{A}$:

$$\mathbf{A} \;=\; \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n \end{bmatrix} \;=\; \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \tag{2.3}$$

The representative agent consumes q_i units of the good produced by firm i. The n-dimensional vector $\mathbf{q}$ contains the quantities of the n firms in the economy and is given by

$$\mathbf{q} \;=\; \begin{bmatrix} q_1 & q_2 & \cdots & q_n \end{bmatrix}' \tag{2.4}$$

An *allocation* is a vector $\mathbf{q}$ that specifies how many units q_i of each good i are produced.

As a consumer the representative agent linearly combines the characteristics of different products. Denote the total units of common characteristic j by x_j

$$x_j \;=\; \sum_i a_{ji} q_i \tag{2.5}$$

The matrix $\mathbf{A}$ converts units of goods purchased $\mathbf{q}$ into units of common characteristics:

$$\mathbf{x} \;=\; \mathbf{A}\mathbf{q} \tag{2.6}$$

Each unit of good i also contains exactly one unit of its own idiosyncratic characteristic. Therefore, we can simply write q_i instead of the units of idiosyncratic characteristic i. The representative agent has a utility function which is quadratic in common ($\mathbf{x}$) and idiosyncratic ($\mathbf{q}$) characteristics

[5] More recently, Pellegrino (2019) and Eeckhout and Veldkamp (2021) use similar hedonic-linear demand systems that model goods as bundles of attributes to study the welfare consequences of market power.

of the products consumed, and suffers a linear disutility for the total number of work hours H. The utility function is therefore given by

$$U(\mathbf{x}, \mathbf{q}, H) \quad \stackrel{\text{def}}{=} \quad \alpha \cdot \sum_{j=1}^{m} \left(b_j^x x_j \; - \; \frac{1}{2} x_j^2 \right) + (1-\alpha) \sum_{i=1}^{n} \left(b_i^q q_i \; - \; \frac{1}{2} q_i^2 \right) - H \tag{2.7}$$

where $\alpha \in [0,1]$ is the utility weight of common characteristics relative to idiosyncratic characteristics. It determines the degree of *horizontal differentiation* between products (Epple, 1987). b_j^x and b_i^q are preference shifters for common and idiosyncratic characteristics.

We close the general equilibrium model by making leisure the outside good. The labor input used by firm i is h_i and thus the labor market clearing condition is given by

$$H \; = \; \sum_i h_i. \tag{2.8}$$

The price of one unit of labor is normalized to 1 such that labor is the numéraire of this economy. Therefore, the total variable cost of firm i is equal to the labor input h_i. Each firm i uses a quasi-Cobb Douglas production function to produce output q_i

$$q_i \; = \; k_i^{\theta} \cdot \ell(h_i) \tag{2.9}$$

where k_i is the (fixed) capital input. The function $\ell(\cdot)$ is such that firm i's production technology can be represented by a quadratic total variable cost function

$$h_i \; = \; c_i q_i \; + \; \frac{\delta_i}{2} q_i^2 \tag{2.10}$$

where c_i and δ_i depend on k_i. The marginal cost (MC) and the average variable cost (AVC) are therefore given by

$$\text{MC}_i \; = \; c_i + \delta_i q_i \qquad \text{and} \qquad \text{AVC}_i \; = \; c_i + \frac{\delta_i}{2} q_i. \tag{2.11}$$

The representative agent purchases and consumes the goods bundle $\mathbf{q}$ taking prices $\mathbf{p}$ as given and receives the aggregate profits from holding shares of all the companies in the economy. We specify the exact ownership arrangements in Section 2.4. The agent's budget constraint is thus given by

$$H + \Pi \; = \; \sum_{i=1}^{n} p_i q_i. \tag{2.12}$$

2.2 Consumption Choices, Labor Supply, and Product Demand

Define

$$\mathbf{b} \quad \stackrel{\text{def}}{=} \quad \alpha \, \mathbf{A}' \mathbf{b}^x + (1-\alpha) \, \mathbf{b}^q \tag{2.13}$$

We obtain the Lagrangian for the representative agent by plugging equation (2.6) and (2.13) into equation (2.7):

$$\mathscr{L}(\mathbf{q}, H) \quad = \quad \mathbf{q}'\mathbf{b} - \frac{1}{2}\mathbf{q}'\left[\mathbf{I} + \alpha\left(\mathbf{A}'\mathbf{A} - \mathbf{I}\right)\right]\mathbf{q} - H - \lambda\left(\mathbf{q}'\mathbf{p} - H - \Pi\right) \tag{2.14}$$

Labor is the numéraire and hence the Lagrange multiplier is $\lambda = 1$. As a result, the consumer's demand function $\mathbf{q}(\mathbf{p})$ maximizes the consumer surplus function:

$$\mathrm{CS}(\mathbf{q}) \quad = \quad \mathbf{q}'(\mathbf{b} - \mathbf{p}) - \frac{1}{2}\mathbf{q}'\left[\mathbf{I} + \alpha\left(\mathbf{A}'\mathbf{A} - \mathbf{I}\right)\right]\mathbf{q} \tag{2.15}$$

$\mathbf{a}_i'\mathbf{a}_j$ is the *cosine similarity* between i and j and ranges from 0 to 1.[6] The *cosine similarities* between all firm pairs are contained in the matrix $\mathbf{A}'\mathbf{A}$. When two products overlap more in the characteristics space, they have a higher cosine similarity which is reflected in the product substitution patterns. If $\mathbf{a}_i'\mathbf{a}_j > \mathbf{a}_i'\mathbf{a}_{j'}$, an increase in the supply of product i leads to a larger decrease in the marginal utility of product j than of product j'.

Define the following matrix

$$\mathbf{\Sigma} \stackrel{\text{def}}{=} \alpha\left(\mathbf{A}'\mathbf{A} - \mathbf{I}\right). \tag{2.16}$$

and let its entries be denoted by σ_{ij}. Thus, the demand and inverse demand functions are

$$\text{Aggregate demand}: \quad \mathbf{q} = (\mathbf{I} + \mathbf{\Sigma})^{-1}(\mathbf{b} - \mathbf{p}) \tag{2.17}$$

$$\text{Inverse demand}: \quad \mathbf{p} = \mathbf{b} - (\mathbf{I} + \mathbf{\Sigma})\mathbf{q} \tag{2.18}$$

The production decision of each firm affects the prices of all the other products in the economy. Specifically, the derivative $\partial p_i/\partial q_j$ is proportional to the product similarity $\mathbf{a}_i'\mathbf{a}_j$. The closer firms i and j are in terms of their product characteristics, the larger this derivative is in absolute value. Because of the symmetry of $\mathbf{A}'\mathbf{A}$, we have $\partial q_i/\partial p_j = \partial q_j/\partial p_i$.

Finally, the profits π_i of firm i are given by

$$\begin{aligned}\pi_i(\mathbf{q}) \quad &\stackrel{\text{def}}{=} \quad p_i(\mathbf{q}) \cdot q_i - h_i \\ &= \quad q_i(b_i - c_i) - \left(1 + \frac{\delta_i}{2}\right)q_i^2 - \sum_{j \neq i}\sigma_{ij}q_iq_j.\end{aligned} \tag{2.19}$$

[6] We use this geometric terminology because $\mathbf{a}_i'\mathbf{a}_j$ measures the cosine of the angle between vectors $\mathbf{a}_i$ and $\mathbf{a}_j$ in the space of common characteristics $\mathbb{R}^m$.

2.3 Advantages of GHL

2.3.1 Markups and Price-Cost Passthrough

First, the GHL demand system differs from the more standard CES demand setup in the markups it produces. GHL produces heterogeneous markups because, unlike CES, the own demand elasticity is not constant and the cross demand elasticities are different across product pairs.

Second, because GHL produces linear (as opposed to isoelastic) residual demands, demand elasticity decreases with firm size and bigger firms charge larger markups. Because we are interested in the quantitative welfare implications of oligopoly and common ownership and these crucially rely on our demand estimates, it is particularly important that the demand system of our model provides a good approximation of actual demand. Thus, an important issue to evaluate is how well GHL matches empirical demand curves estimates when compared to CES.

In Figure 1 we plot three different demand curves based on CES demand, GHL demand, and non-parametric demand estimates by Baqaee and Farhi (2020a) respectively.[7] Whereas the GHL demand curve closely matches the non-parametric estimates of Baqaee and Farhi (2020a), the isoelastic CES demand curve does not seem to match the data very well.

2.3.2 Complementarities

Our network Cournot model allows for complementarities. This is despite the fact that $\mathbf{\Sigma}$ is non-negative by construction and hence the marginal utility from one unit of product j is always non-increasing in q_i:

$$\frac{\partial^2 \text{CS}}{\partial q_i \partial q_j} \;=\; -\sigma_{ij} \;\leq\; 0 \quad \forall\, i \neq j \tag{2.20}$$

However, the non-negativity of $\mathbf{\Sigma}$ does not mean that all products are substitutes and that no pair of products are complements. Recall the definition of complements and substitutes based on cross-price effects:

$$\text{Complements} \quad \text{if} \quad \frac{\partial q_i}{\partial p_j} < 0 \qquad \text{Substitutes} \quad \text{if} \quad \frac{\partial q_i}{\partial p_j} > 0 \tag{2.21}$$

The important insight is that the cross-price elasticity of demand depends on the inverted matrix $(\mathbf{I} + \mathbf{\Sigma})^{-1}$, not on $\mathbf{\Sigma}$ itself. If, as in our case, the off-diagonal elements of $\mathbf{\Sigma}$ are not equal to each other, $-(\mathbf{I} + \mathbf{\Sigma})^{-1}$ will generally include positive as well as negative elements. This implies that some product ik pairs are complements in the sense defined above and thus the quantity choices q_i and q_k can be strategic complements. Intuitively, this complementarity arises from the fact that, in our model, "the enemy of my enemy is my friend." An increase in quantity q_i leads to a reduction in

[7] Baqaee and Farhi (2020a) fit a residual demand curve non-parametrically using price-cost passthrough estimates by Amiti, Itskhoki and Konings (2019) based on Belgian manufacturing enterprise micro data.

FIGURE 1: DEMAND SYSTEM COMPARISON

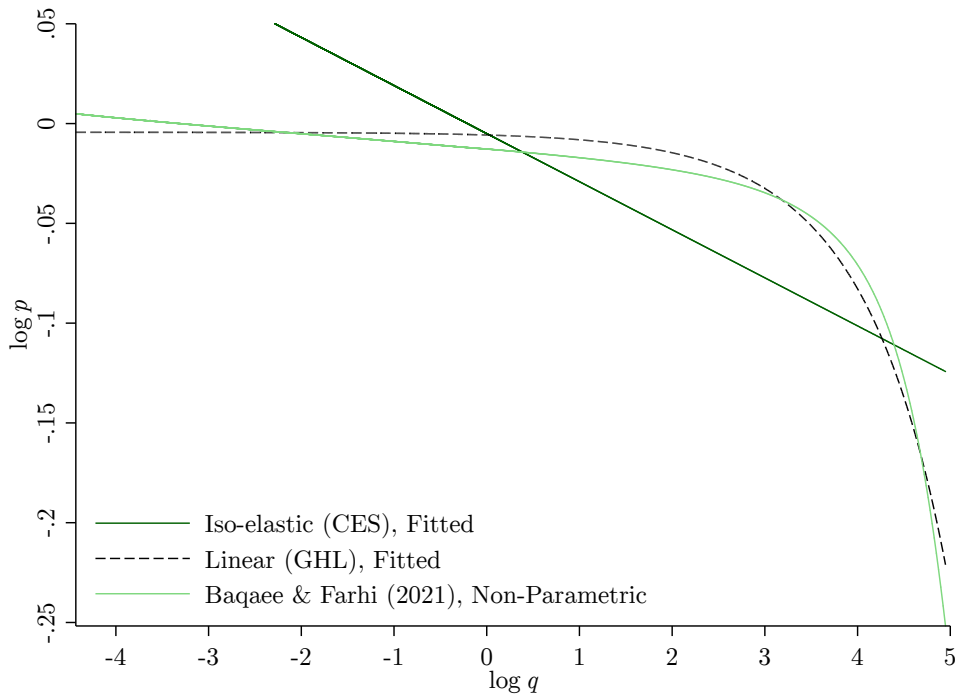

FIGURE NOTES: The figure shows a log-log plot of the demand curve estimated non-parametrically by Baqaee and Farhi (2020a) (solid light green line), against a linear (GHL) fit (dotted black line) and against an isoelastic (CES) fit (solid dark green line). Note that an isoelastic curve becomes a straight line in a log-log plot. Baqaee and Farhi (2020a)'s model demand curve is obtained from price-cost passthrough estimates by Amiti et al. (2019).

residual demand for firm j and thus a decrease in quantity q_j, but this in turn implies an increase in residual demand firm k and thus an increase in quantity q_k.

This complementarity matches realistic features of economy-wide substitution patterns. For example, our computed vector of cross-price derivatives for General Motors in 2018 includes several negative elements (i.e., complements), including energy and consumer finance companies: higher oil prices, loan rates, or insurance premia adversely affect the residual demand for cars.

2.4 Ownership and the Firms' Objective

There are Z investment funds which are owned by the representative agent and indexed by z. V_z, the value of fund z, is the sum of the profits that they are entitled to based on their ownership share in each company i

$$V_z \overset{\text{def}}{=} \sum_{i=1}^{n} s_{iz}\,\pi_i \qquad \text{and} \qquad \sum_{z=1}^{Z} s_{iz} = 1 \tag{2.22}$$

where s_{iz} is the proportion of shares of company i owned by investor z.[8] Following Rotemberg (1984), we assume that the manager of firm i maximizes ϕ_i, which is the sum of all investors' value functions, weighted by their respective ownership shares in firm i:

$$\phi_i \overset{\text{def}}{=} \sum_{z=1}^{Z} s_{iz} V_z = \sum_{z=1}^{Z} s_{iz} \sum_{j=1}^{n} s_{jz} \pi_j = \sum_{j=1}^{n} \pi_j \sum_{z=1}^{Z} s_{iz} s_{jz} \tag{2.23}$$

which we are able to rewrite on the right-hand side as a weighted sum of all firms' profits by substituting V_z from equation (2.22).

We assume that firms engage in Cournot competition and that the profit functions are concave. Hence, to maximize V_i, firm i's management sets the following derivative with respect to q_i equal to zero:

$$\begin{aligned} \frac{\partial \phi_i}{\partial q_i} &= \sum_{j=1}^{n} \mathbf{s}_i' \mathbf{s}_j \cdot \frac{\partial \pi_j}{\partial q_i} \\ &= \mathbf{s}_i' \mathbf{s}_i \left[b_i - c_i - (2 + \delta_i) q_i - \alpha \sum_{j \neq i} \mathbf{a}_i' \mathbf{a}_j q_i \right] - \alpha \sum_{j \neq i} \mathbf{s}_i' \mathbf{s}_j \cdot \mathbf{a}_i' \mathbf{a}_j \cdot q_i \end{aligned} \tag{2.24}$$

where

$$\mathbf{s}_i \overset{\text{def}}{=} \begin{bmatrix} s_{i1} & s_{i2} & \dots & s_{iZ} \end{bmatrix}' \tag{2.25}$$

Define the common ownership weights κ_{ij} as

$$\kappa_{ij} \overset{\text{def}}{=} \frac{\mathbf{s}_i' \mathbf{s}_j}{\mathbf{s}_i' \mathbf{s}_i}. \tag{2.26}$$

This allows us to rewrite firm i's objective function in the following way

$$\phi_i \propto \pi_i + \sum_{j \neq i} \kappa_{ij} \pi_j. \tag{2.27}$$

Our notation follows Backus et al. (2021b) and Antón et al. (2020).[9] We interpret κ_{ij} as the weight—due to common ownership—that each firm (or each manager) i's objective function assigns to the profits of other firms relative to its own profits and corresponds to what Edgeworth (1881) termed the "coefficient of effective sympathy among firms."

At this point it is worth discussing our assumption that the manager of firm i maximizes ϕ_i.

[8] We assume that ownership allocations are exogenous as is standard in the literature. This assumption is further justified by recent theoretical work on endogenous ownership by Piccolo and Schneemeier (2020). They predict that, in the presence of noise traders and investment costs, common ownership will arise through investors' endogenous trading in a way that is unpredictable based on the structure of product markets (i.e., there are multiple equilibria with varying degrees of common ownership).

[9] López and Vives (2019) and Azar and Vives (2021a) use the same formulation but denote κ_{ij} by λ_{ij}.

First, there is a long tradition in economics of weighting shareholder interests in the objective function of the firm, including Drèze (1974), Grossman and Hart (1979), and Rotemberg (1984). Second, more recently, the common ownership literature has used the same objective function for firms as in equation (2.27) with Azar (2020) providing microeconomic foundations for the firm manager's maximization choice.

However, this assumption that firms (or managers) maximize the weighted portfolio profits of their investors differs from Azar and Vives (2021a). They instead assume a two-class economy with worker-consumers and owner-consumers, in which firms maximize the weighted *utilities* of their owner-consumers.[10] In their model, firms are assumed to take into account that their strategic decisions affect both their investors' portfolio profits *and* their investors' consumption choices through the firm quantities' influence on the aggregate price index. For example, under the latter part of this assumption airlines internalize that some of its investors are also air travelers and setting higher quantities lowers the relative price of air travel in the consumption bundle of these owner-consumers which directly benefits them.

In contrast, in our model firms only internalize all effects on investors' portfolio profits but ignore the impact (of the production quantity choices) on the consumption bundles of their investors. First, we believe that this assumption is a better description of what firms actually do. In an economy with many diverse consumer-owners who consume changing baskets of goods it would be exceedingly difficult for those at the helm of firms to keep track of the impact of strategic decisions on investor utilities. Second, it is likely that the consumption bundles of firm managers and other consumers in the economy differ significantly. For example, Bertrand and Kamenica (2018) document that the divergence in consumer behavior is fairly constant over time and larger than other cultural distances such as media diet, time use, or social attitudes. Assume, therefore, that there are two types of agents: managers and consumer-workers. The strategic decisions of each firm i are in the hands of a manager (also indexed by i). We assume that the manager's compensation is equal to

$$\omega_i = \varepsilon \sum_{z=1}^{Z} s_{iz} V_z \tag{2.28}$$

with ε arbitrarily small. Managers have increasing utility over and spend all of their income on a "luxury" good (e.g., yachts or private jets)[11] indexed by 0, which is produced competitively with a linear technology using only labor. Let q_{i0} denote the quantity of the luxury good purchased by the manager of firm i, p_0 the price, and h_0 the total work. We then have the following market clearing

[10] Oligopoly deadweight losses, markups, and misallocations arise because of a conflict between classes. At a fundamental level, this is because the agents that are in charge of production, consume a different set of goods from the rest, and therefore do not internalize how their choices affect the real income of other agents. In Azar and Vives (2021a) the two-class divide between worker-consumers and owner-consumers is the source of the anticompetitive labor market effect of common ownership.

[11] This setup is reminiscent of Veblen (1899) because these agents engage in (conspicuous) consumption that is distinct from the basket of goods consumed by the representative worker-consumer.

condition for the production of good 0

$$\sum_{i=1}^{I} q_{i0} = h_0 \tag{2.29}$$

and the price p_0 is one by construction. The assumption of ε being arbitrarily small implies that the labor used to produce good 0 is a negligible share of total labor in the economy.

In line with much of the common ownership literature, we focus on unilateral effects (Ivaldi, Jullien, Rey, Seabright and Tirole, 2003b) and do not consider coordinated effects (Ivaldi, Jullien, Rey, Seabright and Tirole, 2003a) of common ownership (e.g., increased incentives and ability to collude). That is to say, we do not assume that firms with overlapping ownership can successfully coordinate their behavior in an anticompetitive way, for example, by lowering quantities.

We can implicitly define a *linear-quadratic network game* (Ballester et al., 2006) by taking the profit vector as a payoff function and the vector of quantities $\mathbf{q}$ as a strategy profile. Loosely speaking, this is because the combination of the matrices $\mathbf{\Sigma}$ (the network of product market rivalry relationships based on the firms' product substitutabilities) and $\mathbf{K}$ (the network of ownership relationships based on the firms' investor shares) is essentially the adjacency matrix of a weighted network. Linear-quadratic network games like ours belong to a larger class of games known as "potential games" (Monderer and Shapley, 1996) because they can be described by a scalar function which is called the game's *potential*.

We can now write the vector of the firms' first order conditions as

$$\mathbf{0} \;=\; (\mathbf{b} - \mathbf{c}) - (2\mathbf{I} + \mathbf{\Delta} + \mathbf{\Sigma} + \mathbf{K} \circ \mathbf{\Sigma})\,\mathbf{q} \tag{2.30}$$

$$\text{where} \qquad \mathbf{\Delta} \overset{\text{def}}{=} \begin{bmatrix} \delta_1 & 0 & \cdots & 0 \\ 0 & \delta_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \delta_n \end{bmatrix} \tag{2.31}$$

The diagonal matrix $\mathbf{\Delta}$ contains the slopes of the firms' marginal cost functions and therefore captures firm-specific economies of scale. $\mathbf{\Sigma}$ and $\mathbf{K}$ are the adjacency matrices of the networks of product rivalry and common ownership. Specifically, $\mathbf{K}$ is the $n \times n$ common ownership matrix of κ_{ij} for all n firms in the economy and $\circ$ denotes the Hadamard (element-by-element) product. $\mathbf{b}$ and $\mathbf{c}$ are the demand and supply function intercepts. b_i is a measure of firm product quality or vertical product differentiation.

Let us define $\Xi(\mathbf{q})$, the potential function of the Cournot game played by the (commonly-owned) firms in our model. By definition, the Cournot-Nash equilibrium allocation under common ownership is the maximizer of $\Xi(\mathbf{q})$. We therefore denote the equilibrium quantity vector by $\mathbf{q}^{\Xi}$. This equilibrium allocation can be found by solving the system of first order conditions in equation (2.30) and is defined as follows:

Definition 1. The *Cournot Common Ownership* (CCO) allocation $\mathbf{q}^{\Xi}$ is defined as the maximizer

of the potential function $\Xi(\mathbf{q})$:

$$\mathbf{q}^{\Xi} \overset{\text{def}}{=} \arg\max_{\mathbf{q}} \Xi(\mathbf{q}) = (2\mathbf{I} + \boldsymbol{\Delta} + \boldsymbol{\Sigma} + \mathbf{K} \circ \boldsymbol{\Sigma})^{-1} (\mathbf{b} - \mathbf{c}). \tag{2.32}$$

2.5 Market Structure and Ownership Counterfactuals

We estimate our theoretical model to analyze how total surplus, profits, and consumer surplus depend on market structure, ownership allocations, and firm conduct. Our baseline assumption is that firms compete as in an economy-wide Cournot oligopoly game in which the manager of each firm i maximizes the objective function ϕ_i which results in the Cournot Common Ownership allocation in equation (2.32). We now consider counterfactual scenarios in which firms make production decisions with alternative objective functions. For example, rather than maximizing portfolio profits ϕ_i firms maximize their own individual firm profits π_i as under standard Cournot competition. Each of these counterfactuals, summarized in the set of equations in (2.33) and inversely ranked by their degree of competitiveness, is the maximizer of a specific scalar quadratic function, which we call *potential* following the nomenclature of Monderer and Shapley (1996).[12] To continue the previous example, the potential function $\Psi(\mathbf{q})$ is the objective function of the pseudo-planner problem for the Cournot-Nash equilibrium allocation without common ownership.

$$\begin{aligned}
\text{Monopoly Potential}: \quad \Pi(\mathbf{q}) &= \mathbf{q}'(\mathbf{b} - \mathbf{c}) && - \mathbf{q}' \left(\mathbf{I} + \frac{1}{2}\boldsymbol{\Delta} + \boldsymbol{\Sigma} \right) \mathbf{q} \\
\text{CCO Potential}: \quad \Xi(\mathbf{q}) &= \mathbf{q}'(\mathbf{b} - \mathbf{c}) && - \mathbf{q}' \left(\mathbf{I} + \frac{1}{2}\boldsymbol{\Delta} + \frac{1}{2}\boldsymbol{\Sigma} + \frac{1}{2}\mathbf{K} \circ \boldsymbol{\Sigma} \right) \mathbf{q} \\
\text{Cournot Potential}: \quad \Psi(\mathbf{q}) &= \mathbf{q}'(\mathbf{b} - \mathbf{c}) && - \mathbf{q}' \left(\mathbf{I} + \frac{1}{2}\boldsymbol{\Delta} + \frac{1}{2}\boldsymbol{\Sigma} \right) \mathbf{q} \\
\text{Total Surplus}: \quad W(\mathbf{q}) &= \mathbf{q}'(\mathbf{b} - \mathbf{c}) && - \frac{1}{2} \cdot \mathbf{q}' (\mathbf{I} + \boldsymbol{\Delta} + \boldsymbol{\Sigma}) \mathbf{q}
\end{aligned} \tag{2.33}$$

We first consider *Cournot* competition which assumes away any common ownership effects by assuming that investors do not hold diversified portfolios.

Definition 2. The *Cournot* allocation $\mathbf{q}^{\Psi}$ is defined as that in which all profit weights κ_{ij} in $\mathbf{K}$ are equal to 0 for $i \neq j$ and equal to 1 for $i = j$:

$$\mathbf{q}^{\Psi} \overset{\text{def}}{=} \arg\max_{\mathbf{q}} \Psi(\mathbf{q}) = (2\mathbf{I} + \boldsymbol{\Delta} + \boldsymbol{\Sigma})^{-1} (\mathbf{b} - \mathbf{c}) \tag{2.34}$$

[12] For our theoretical analysis, we assume an interior solution for the closed-form expressions of $\mathbf{q}$. For our empirical analysis, we also compute a numerical solution with a non-negativity constraint on $\mathbf{q}$. The non-negativity constraint binds for very few firms and the solution is almost identical to the unconstrained solution (e.g., error $< 0.1\%$ for the total surplus function).

Next we consider *Perfect Competition*. In this scenario firms behave as if they are atomistic producers pricing all units at marginal cost.

Definition 3. The *Perfect Competition* allocation $\mathbf{q}^W$ is defined as the maximizer of the aggregate total surplus function $W(\mathbf{q})$:

$$\mathbf{q}^W \quad \stackrel{\text{def}}{=} \quad \arg\max_{\mathbf{q}} W(\mathbf{q}) \quad = \quad (\mathbf{I} + \boldsymbol{\Delta} + \boldsymbol{\Sigma})^{-1}(\mathbf{b} - \mathbf{c}) \tag{2.35}$$

The least competitive allocation is *Monopoly*. In this setting a single investment fund controls the quantity decisions of all firms in the economy and maximizes the firms' aggregate profits without any regard for consumer surplus.

Definition 4. The *Monopoly* allocation $\mathbf{q}^\Pi$ is defined as the maximizer of the aggregate profit function $\Pi(\mathbf{q})$:

$$\mathbf{q}^\Pi \quad \stackrel{\text{def}}{=} \quad \arg\max_{\mathbf{q}} \Pi(\mathbf{q}) \quad = \quad (2\mathbf{I} + \boldsymbol{\Delta} + 2\boldsymbol{\Sigma})^{-1}(\mathbf{b} - \mathbf{c}) \tag{2.36}$$

This allocation obtains in an economy without any antitrust restrictions on ownership allocations and in which firms have unconstrained ability to coordinate their supply decisions. It is the limit of a Cournot equilibrium with common ownership in which all of the profit weights tend to one (i.e., $\kappa_{ij} \to 1$).

3 Data

3.1 Mapping to Data

Table 1 documents how the variables in our model correspond to data, including the sources of these data. Revenues, variables cost, and fixed costs are measured using data from Compustat. We follow (De Loecker et al., 2020, henceforth DEU) in excluding firms with negative revenues or costs of goods sold, or negative gross margin (revenues less COGS). Furthermore, we follow their computation of the user cost of capital, which is equal to the federal funds rate (FEDFUNDS from FRED), minus capital goods inflation (PIRIC from FRED), plus a combined depreciation rate and risk premium set at 12%.

To construct the networks of product market rivalry and ownership we use data from 10-K product descriptions and 13(f) filings. We discuss the construction of these networks in detail below.

3.2 Text-Based Product Similarity

The matrix of product similarities $\mathbf{A}'\mathbf{A}$ is one of two crucial inputs that are required to estimate our model. (Hoberg and Phillips, 2016, henceforth HP) provide an empirical estimate of this input comes. They compute time-varying product cosine similarities for firms in Compustat by analyzing

TABLE 1: VARIABLE DEFINITIONS AND MAPPING TO DATA

Panel A: Observed Variables

Notation	Concept	Measurement
$p_i q_i$	Revenues	Revenues (*source*: *Compustat*)
h_i	Total Variable Costs	Costs of Goods Sold (*source*: *Compustat*)
$\mathbf{a}_i' \mathbf{a}_j$	Product Cosine Similarity	Word cosine similarity in 10-K product description (*source*: *Hoberg and Phillips, 2016*)
$\mathbf{s}_i$	Ownership	Number of shares divided by total shares outstanding (*source*: *SEC 13(f) filings, Compustat*)

Panel B: Identified Variables

Notation	Derived Variable	Computation/Identification
q_i	Output	$\mathbf{q}$ such that $\pi = \text{diag}(\mathbf{q})\left(\mathbf{I} + \frac{1}{2}\mathbf{\Delta} + \mathbf{K} \circ \mathbf{\Sigma}\right)\mathbf{q}$
c_i	Marginal Cost Intercept	$c_i = \frac{h_i}{q_i} - \frac{1}{2}\delta_i q_i$
$\mathbf{b}$	Demand Intercept	$\mathbf{b} = (2\mathbf{I} + \mathbf{\Delta} + \mathbf{\Sigma} + \mathbf{K} \circ \mathbf{\Sigma})\,\mathbf{q} + \mathbf{c}$

the text of the firms' 10-K forms. Every 10-K form contains a *business description* section, which is the target of the algorithm devised by HP. HP build a vocabulary of 61,146 words that firms use to describe their products, and that identify product characteristics. For each firm i, HP use this vocabulary to construct a vector of word occurrences $\mathbf{o}_i$:

$$\mathbf{o}_i = \begin{bmatrix} o_{i,1} \\ o_{i,2} \\ \vdots \\ o_{i,61146} \end{bmatrix} \tag{3.1}$$

HP normalize this vector by dividing by the Euclidean norm which yields the empirical counterpart of $\mathbf{a}_i$

$$\mathbf{a}_i = \frac{\mathbf{o}_i}{\|\mathbf{o}_i\|}. \tag{3.2}$$

All $\mathbf{a}_i$ vectors are dot-multiplied to obtain $\mathbf{A'A}$:

$$\mathbf{A'A} = \begin{bmatrix} \mathbf{a}_1'\mathbf{a}_1 & \mathbf{a}_1'\mathbf{a}_2 & \cdots & \mathbf{a}_1'\mathbf{a}_n \\ \mathbf{a}_2'\mathbf{a}_1 & \mathbf{a}_2'\mathbf{a}_2 & \cdots & \mathbf{a}_2'\mathbf{a}_n \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{a}_n'\mathbf{a}_1 & \mathbf{a}_n'\mathbf{a}_2 & \cdots & \mathbf{a}_n'\mathbf{a}_n \end{bmatrix} \tag{3.3}$$

To the extent that the word frequencies in the vocabulary constructed by HP correctly represent product characteristics, this matrix is the exact empirical counterpart to $\mathbf{A'A}$—the matrix of cross-price effects in our theoretical model. Because each U.S. publicly listed firm must file a 10-K form the matrix of estimated product similarities provides a singularly comprehensive description of competitive interactions. It also covers the near entirety (97.8%) of the CRSP-Compustat universe.[13]

3.3 Ownership Data

In order to calculate the matrix of common ownership profit weights $\mathbf{K}$, we require the matrix of ownership shares $\mathbf{S}$. We obtain $\mathbf{S}$ from two datasets of mutual fund holdings reported in form 13(f) filings. Form 13(f) is a mandatory filing of the Securities and Exchange Commission (SEC) in which institutional investors with assets under management (AUM) in excess of $100 million are required to report their holdings of U.S. securities, including those of all U.S. public corporations.

[13] One of HP's objectives in developing this dataset is to remedy two well-known shortcomings of the traditional industry classifications: (i) the inability to capture imperfect substitutability between products, which is the most salient feature of our model; and (ii) the fact that commonly used industry classifications, such as SIC and NAICS, are based on similarity in *production processes*, rather than in product characteristics. In other words, they are appropriate for estimating production functions, but unsuitable for proxying for the elasticity of substitution between different products.

Our data covers the period 1994-2018. For the years 1999-2017 we use a dataset constructed by Backus et al. (2021b), who parsed the data contained in 13(f) forms. For the remaining years, we use 13(f) data from Thomson Reuters, obtained through the WRDS platform. We merge this data, using CUSIP codes, to the total amount of shares outstanding provided, for each firm, in Compustat.

By dividing the shareholdings of individual investors by the total number of shares outstanding, we obtain the normalized shares vector $\mathbf{s}_i$. We then apply equation (2.26) to compute the matrix $\mathbf{K}$ of ownership shares.

Finally, we estimate the matrix of profit weights $\mathbf{K}$ using a statistical correction to account for the presence of unobserved investors, as detailed in Appendix A. Constructing detailed ownership data, even for the largest publicly listed U.S. companies such as the constituents of the S&P500, is not without obstacles as noted by Backus et al. (2021b). This process is substantially more difficult for the entire universe of publicly listed U.S. firms. Without such a correction to account for the presence of unobserved investors we would obtain a thick right tail of implausibly large κ_{ij}.

3.4 Calibration and Identification

There are two crucial steps required to take our model to the data and to perform counterfactual analysis. First, we need to calibrate the parameter α and the diagonal matrix $\mathbf{\Delta}$. Second, we have to identify the quantity vector $\mathbf{q}$, the price vector $\mathbf{p}$, the cost intercepts $\mathbf{c}$, and finally the demand intercepts $\mathbf{b}$.

First, following equation (2.10) we note that the cost function must take the form

$$h_i \quad = \quad \tilde{c}_i \left(\frac{q_i}{k_i^\theta} \right) + \frac{\tilde{\delta}}{2} \left(\frac{q_i}{k_i^\theta} \right)^2 \tag{3.4}$$

where, by definition, we have

$$c_i \; = \; \frac{\tilde{c}_i}{k_i^\theta} \qquad \text{and} \qquad \delta_i \; = \; \frac{\tilde{\delta}}{k_i^{2\theta}}. \tag{3.5}$$

We assume that $\tilde{\delta}$ is constant across firms and over time thereby reducing the dimensionality of the cost parameter vector to one.

We take the calibrated parameters for $\alpha = 0.05$ and $\tilde{\delta} = 6.3$ from Pellegrino (2019) and for $\theta = 0.15$ from De Loecker et al. (2020). We check that our model produces an equally good fit of markups and cross-price elasticity data. In Appendix C we document how well our model matches the non-targeted moments.

To identify $\mathbf{q}$ we use the fact that we can write the vector of profits in terms of the quantities

$\mathbf{q}$, and the matrices $(\mathbf{\Delta}, \mathbf{\Sigma}, \mathbf{K})$, which are already measured or calibrated:

$$\pi \;=\; \text{diag}\left(\mathbf{q}\right)\left(\mathbf{I} + \frac{1}{2}\mathbf{\Delta} + \mathbf{K} \circ \mathbf{\Sigma}\right)\mathbf{q} \tag{3.6}$$

While the equation above does not yield a closed-form solution, we can solve for $\mathbf{q}$ numerically. Prices p_i are then obtained by dividing revenues $p_i q_i$ by output q_i. Finally, the demand and marginal cost intercepts are identified as follows:

$$c_i \;=\; \frac{h_i}{q_i} - \frac{\delta}{2} q_i; \qquad \mathbf{b} \;=\; \left(2\mathbf{I} + \mathbf{\Delta} + \mathbf{\Sigma} + \mathbf{K} \circ \mathbf{\Sigma}\right)\mathbf{q} + \mathbf{c} \tag{3.7}$$

4 Empirical Results

Our empirical analysis proceeds in two steps. First, we describe the salient features of the data on product similarity and common ownership. Second, we report the empirical model estimates of welfare, consumer surplus, and profit and their evolution over time.

4.1 Product Similarity and Common Ownership

4.1.1 Network Structure of Product Similarity and Common Ownership

We begin our empirical analysis by visualizing the two networks that characterize our oligopoly game: that of product similarities and that of common ownership. We first visualize the network structure of HP's product similarity dataset. We employ the widely-used network visualization algorithm of (Fruchterman and Reingold, 1991, henceforth FR) to reduce the network's dimensionality from 61,146 (the number of words in the HP's vocabulary) to two (a bidimensional surface). The FR algorithm models the network nodes as particles and arranges them on a plane. However, the algorithm is sensitive to the initial configuration of nodes and has difficulties visualizing the cluster structure of large networks. We address this well-known problem by pre-arranging the nodes with the OpenOrd algorithm (Martin, Brown, Klavans and Boyack, 2011) which was specifically developed for this purpose, before running the FR algorithm.

Every publicly traded firm in 2004 is a dot in each of the two panels of Figure 2. In the left panel, firm pairs with a high cosine product similarity are shown closer and are connected by a thicker line. Two patterns are particularly noteworthy. First, firms are unevenly distributed over the space of product characteristics. In some areas in the left panel of Figure 2 there is a significantly denser population of firms than in other areas. Second, the product similarity network exhibits a distinct community structure: large groups of firms cluster in the same areas of the network.

We repeat the same exercise for the network of ownership links between all the companies in our sample. As before, we reduce the dimensionality of the dataset from 3,126 (the number of investors) to two using the OpenOrd and FR algorithms to visualize the network in the right panel

FIGURE 2: NETWORK VISUALIZATION

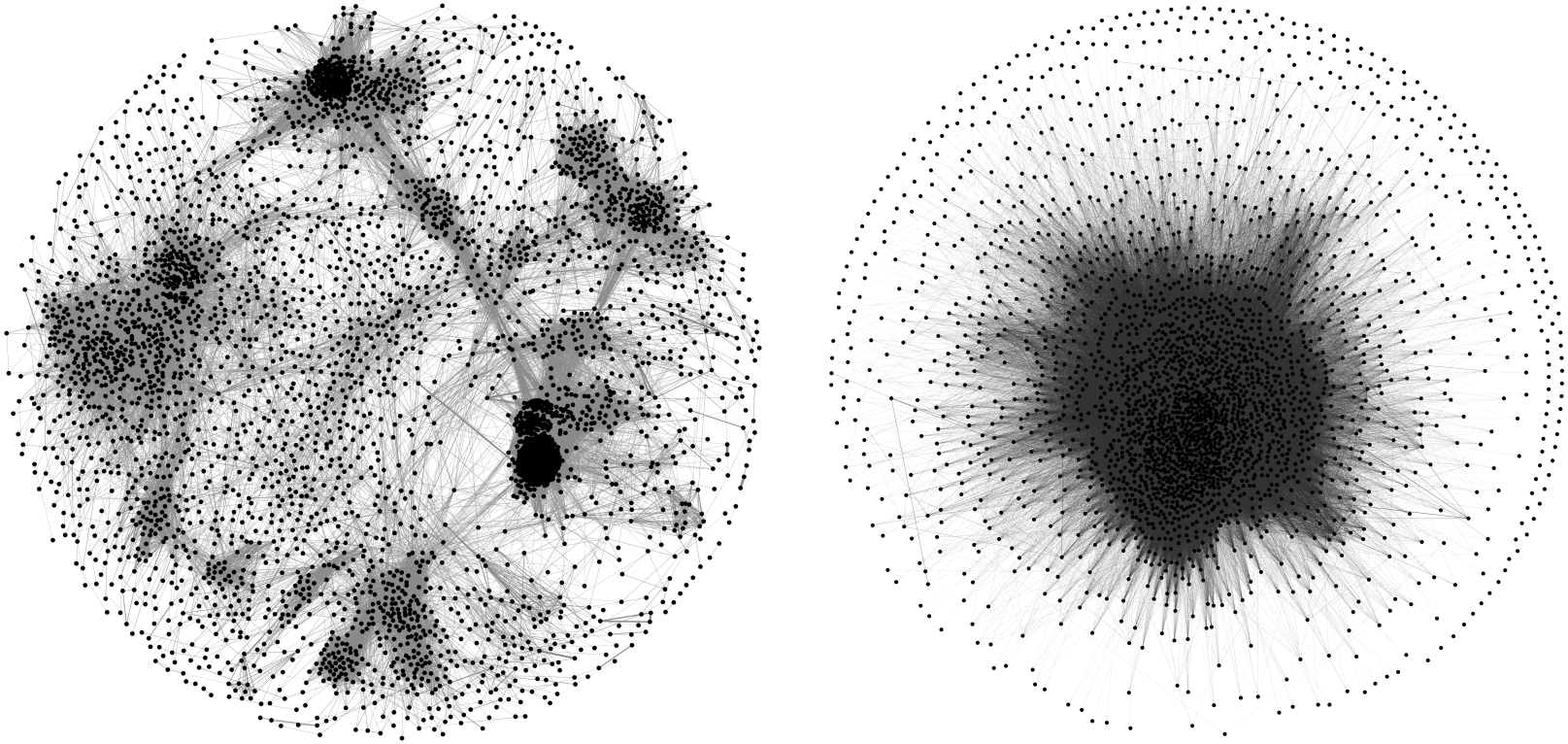

FIGURE NOTES: The diagrams are two-dimensional representations of the network of product similarities (left panel), first computed by Hoberg and Phillips (2016), and of the network of ownership shares (right panel). The data cover the universe of publicly listed firms in 2004. Firm pairs that are closer in product market space and closer in ownership space, are shown with thicker links. These distances are computed in spaces that have approximately 61,000 and 3,100 dimensions, respectively. We use the OpenOrd algorithm of Martin et al. (2011) and the FR gravity algorithm of Fruchterman and Reingold (1991) to plot these high-dimensional objects over a plane.

of Figure 2. Firm pairs that have large ownership weights between them are shown closer and are connected by a thicker line. In contrast to the product similarity network depicted in Figure 2 the network does not exhibit a community structure, but instead has a distinct hub-and-spoke structure with a large proportion of firms sharing significant overlap and a remainder of largely unconnected firms at the periphery.

Because both networks are based on time-varying relationships between the different constituents of the universe of public companies, they are not static networks, but evolve over the course of our study period. However, the two networks differ markedly in their evolution over time. The network of product similarity does not change much as measured by the average value of $\mathbf{a}_i'\mathbf{a}_j$, which is equal to 0.0171 in 1994 and slightly increases to 0.0174 in 2018. In contrast, the average value of κ_{ij} is equal to 0.0021 in 1994 but rises almost sevenfold to 0.0146 in 2018.

4.1.2 Relationship between Product Similarity and Common Ownership

A crucial aspect of our empirical analysis is to document the empirical relationship between product similarity $\mathbf{\Sigma}$ and common ownership $\mathbf{K}$ because this relationship governs the magnitude of the welfare cost of common ownership. As can be seen from equation (2.32) it is the Hadamard product of $\mathbf{K}$ and $\mathbf{\Sigma}$ that determines how much the realized quantity choices of firms under Cournot competition with common ownership differ from the standard benchmarks of standard Cournot without common ownership in equation (2.34) and monopoly in equation (2.36).

Figure 3 plots the histogram of the joint distribution of the product similarity $\mathbf{a}_i \cdot \mathbf{a}_j$ and the common ownership weight κ_{ij} for any firm pair i and j in 2018. Although each product similarity pair $\mathbf{a}_i \cdot \mathbf{a}_j$ is symmetric, the common ownership weight κ_{ij} is not symmetric. We therefore plot each pair of firm i and j twice.

A large proportion of firm pairs has little product similarity and little common ownership between them. The complete absence of overlap is relatively more pronounced in ownership than in product similarity space as evidenced by the discontinuous jump at 0 for κ_{ij}. However, a sizable proportion of firm pairs overlaps considerably in both product similarity and ownership space. There is no clear relationship between product similarity and common ownership. The correlation between the two variables in 2018 is 0.0034. This means that common ownership is not much more pronounced for firms that are more similar in product space.

Finally, the figure also shows that a small proportion of κ_{ij} has values greater than 1. Such values of κ exceeding 1 lead to owners placing more weight on the profits of competitor j than on the profits of their own firm i. This makes it possible for common ownership to create incentives for the "tunneling" of profits from one firm to another (Johnson, La Porta, Lopez-de Silanes and Shleifer, 2000). However, the proportion of these firms is sufficiently small such that even if we restrict all κ_{ij} to be strictly smaller than 1, the estimates of our model are essentially unchanged.

FIGURE 3: PRODUCT SIMILARITY AND PROFIT WEIGHTS (2018)

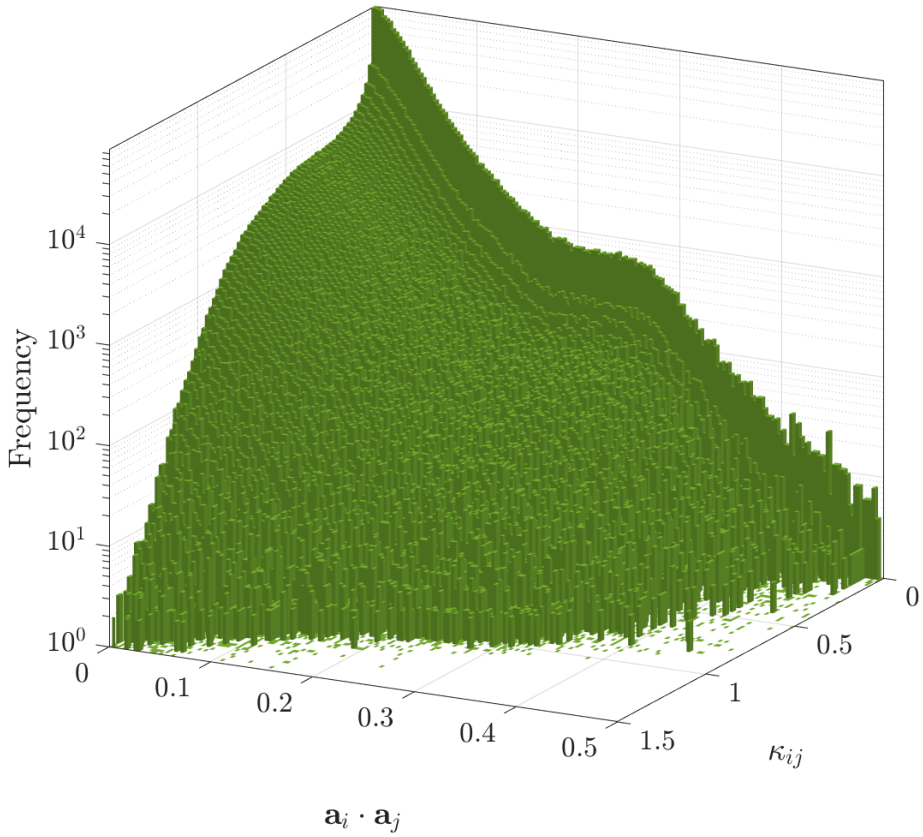

FIGURE NOTES: The figure above is a histogram representing of the joint distribution of product similarity $\mathbf{a}_i \cdot \mathbf{a}_j$ and profit weights κ_{ij} for all firm ij pairs in 2018.

4.2 Welfare, Consumer Surplus, and Profit Estimates

We now present the results of the empirical estimation of our model. These baseline estimates assume that investors exert influence in proportion to their ownership shares and that firms set quantities in accordance with the objective function given in equation (2.23).

We first compute total surplus and decompose it into profits and consumer surplus as reported in Table 2 for 2018, the most recent year in our sample. These calculations are based on the assumption that the observed equilibrium is the Cournot-Nash equilibrium under common ownership (column 1) of our model in Section 2. In columns 2, 3, and 4 we report the counterfactual estimates based on the alternative model assumptions. Table 3 in the appendix reports the same estimates for 1994, the first year of our sample.

We estimate that in 2018, under *Common Ownership*, publicly listed firms earn an aggregate economic profit of \$5.639 trillion, consumers gain a surplus of \$3.314 trillion, and the estimated total surplus is equal to \$8.953 trillion. 63% of the total surplus produced accrues to companies in

the form of oligopoly profits under common ownership while consumers appropriate the remaining 37% of total surplus.[14]

The estimates for our two primary counterfactuals, *Cournot-Nash* and *Perfect Competition*, are reported in column 2 and 3. Comparing the estimates of these counterfactual models with those of the *Common Ownership* allocation in column 1 shows that the welfare costs of common ownership are significant, but not as large as the welfare costs of oligopoly. First, total surplus is slightly higher at $9.374 trillion under oligopoly without common ownership (Cournot-Nash) and substantially higher at $10.597 trillion under perfect competition. Thus, we estimate that in 2018 the deadweight loss of oligopoly amounts to about 11.5% of total surplus. On top of that, common ownership leads to an additional deadweight loss of 4% of total surplus.

Although the effects of oligopoly and common ownership on efficiency are considerable, their respective distributional effects are even more substantial. Under perfect competition consumers capture a much larger share of the total surplus: $8.565 trillion, more than double than in the Cournot-Nash ($4.113 trillion) and the Common Ownership ($3.314 trillion) allocations. This means that when firms price at marginal cost 80.8% of the total surplus accrues to consumers. In contrast, merely 43.9% and 37% of total surplus accrue to consumers under oligopoly without and with common ownership. Corporate profits, on the other hand, move exactly in the opposite direction. The aggregate profits under common ownership ($5.639 trillion) are almost 3 times as large as those under perfect competition ($2.033 trillion).

The comparison between *Common Ownership* in column 1 and *Cournot-Nash* in column 2 further allows us to focus on the distributional effects of common ownership on top of the effect of product market power due to oligopoly. Not only does common ownership in the economy lead to a total welfare loss of $421 billion, but the welfare losses of common ownership fall entirely on consumers. Common ownership raises lowers consumer surplus by $799 billion from $4.113 trillion to $3.314 trillion.

In contrast, common ownership raises aggregate profits by $378 billion from $5.261 trillion to $5.639 trillion. This aggregate increase in corporate profits however obscures the fact that common ownership differentially affects corporate profits, as can be seen in Table 4 in the appendix which lists the companies that experience the largest profit increases and decreases due to common ownership. Although the vast majority of companies (98.04%) has higher profits under common ownership, a small minority (1.96%) earns lower profits. This differential impact of common ownership occurs for several reasons. First, as documented in Figure 2 and Figure 3 there is a great deal of heterogeneity in common ownership. Second, the magnitude of the impact of common ownership depends on companies' position in the network of product market rivalry. Third, when

[14] It is instructive to put these estimates into context by comparing them to the GDP of U.S. corporations in the same year which is around $11 trillion. There are two differences between the total surplus computed here and GDP: the value of labor input (not included in our total surplus, but included in GDP) and the value of inframarginal consumption (included in our total surplus, but not included in GDP).

TABLE 2: WELFARE ESTIMATES (2018)

	Scenario	*Common Ownership*	*Cournot-Nash*	*Perfect Competition*	*Monopoly*
		(1)	(2)	(3)	(4)
Welfare Statistic	**Variable**	$\mathbf{q}^{\Xi}$	$\mathbf{q}^{\Psi}$	$\mathbf{q}^{W}$	$\mathbf{q}^{\Pi}$
Total Surplus (US$ trillions)	$W(\mathbf{q})$	8.953	9.374	10.597	8.484
Aggregate Profits (US$ trillions)	$\Pi(\mathbf{q})$	5.639	5.261	2.033	5.878
Consumer Surplus (US$ trillions)	$\mathrm{CS}(\mathbf{q})$	3.314	4.113	8.565	2.606
Total Surplus / Perfect Competition	$\frac{W(\mathbf{q})}{W(\mathbf{q}^{W})}$	0.845	0.885	1.000	0.802
Aggregate Profit / Total Surplus	$\frac{\Pi(\mathbf{q})}{W(\mathbf{q})}$	0.630	0.561	0.192	0.693
Consumer Surplus / Total Surplus	$\frac{\mathrm{CS}(\mathbf{q})}{W(\mathbf{q})}$	0.370	0.439	0.808	0.307

TABLE NOTES: The table reports the model estimates of aggregate profits, consumer surplus, and total surplus for each of the counterfactual scenarios presented in Section 2.

FIGURE 4: TOTAL SURPLUS OF U.S. PUBLIC FIRMS

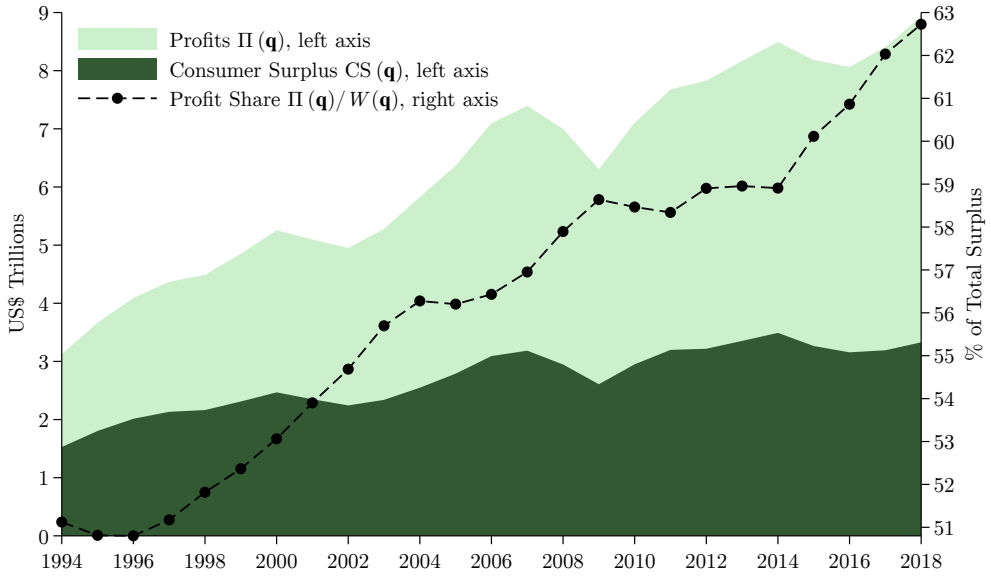

FIGURE NOTES: The figure plots the evolution of aggregate (economic) profits $\Pi(\mathbf{q})$, aggregate consumer surplus $CS(\mathbf{q})$, and total surplus $W(\mathbf{q})$ using the left axis. The right axis shows profits as a percentage of total surplus $(\Pi(\mathbf{q})/W(\mathbf{q})$, black dotted line).

common ownership increases, all firms produce less in the aggregate (raising profits overall), but there is also a reallocation of market shares towards more productive firms. For some unproductive firms the effect of reallocation on profits is negative and sufficiently strong to counteract the broad profit-increasing effects of common ownership. Thus, it is not surprising that some of the largest and most profitable firms in the U.S. economy benefit the most from common ownership relative to the uncoordinated Cournot oligopoly benchmark.

The final counterfactual we analyze is the *Monopoly* allocation for which we report the welfare estimates in column 4. Recall that under this allocation all firms are controlled by a single decision-maker who coordinates supply choices and maximizes aggregate firm profits. Aggregate surplus is equal to only \$8.484 trillion and thus significantly lower than in the common ownership equilibrium allocation. Despite the decrease in aggregate welfare, profits are markedly higher still at \$5.878 trillion. In contrast, consumer surplus is reduced to just \$2.606 trillion equal to 30.7% of the total surplus.[15]

The broad conclusion of our comparative static analysis is that the combination of oligopoly and common ownership of U.S. public firms has considerable effects on allocative efficiency, firm profits, and consumer welfare.

[15]Table 5 in the appendix reports the welfare estimates for 2018 for the case in which all κ_{ij} are restricted to be smaller than 1. Although the overall levels of our estimates are slightly different, these estimates lead to almost identical results for the effect of monopoly, oligopoly, and common ownership on total surplus, aggregate profit, and consumer surplus.

FIGURE 5: PROFIT SHARE OF TOTAL SURPLUS

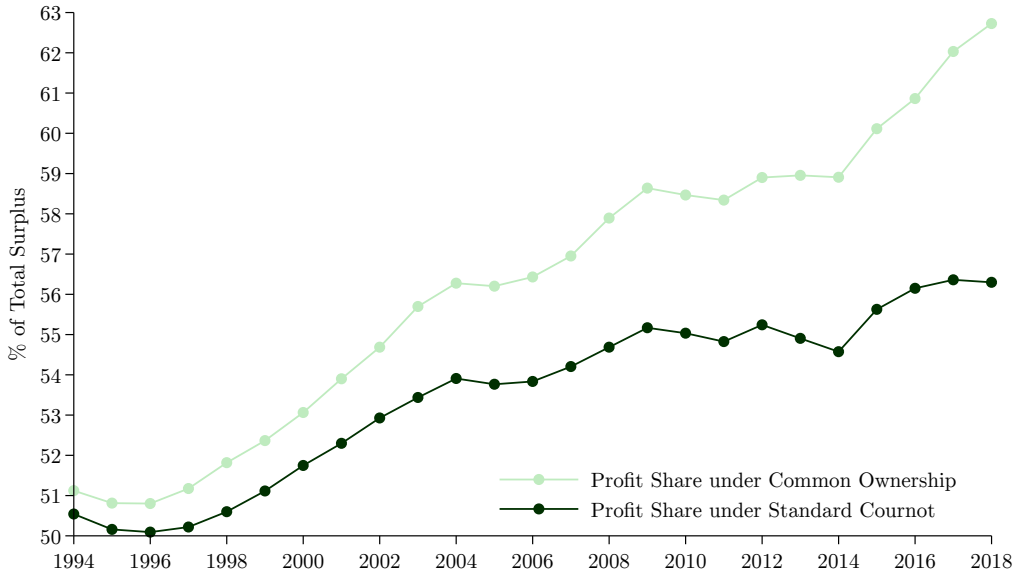

FIGURE NOTES: The figure plots the profit share under standard Cournot (dark green line) and Cournot with common ownership (light green line) between 1994 and 2018.

4.3 Time Trends in Welfare, Consumer Surplus, and Profits

We now consider time trends in welfare, consumer surplus, and firm profits based on annual estimates obtained from mapping our model to Compustat data on a yearly basis. We are particularly interested in the welfare implications of the joint rise of product market and ownership concentration among publicly listed U.S. companies for the period from 1994 to 2018.[16]

In Figure 4, we plot annual aggregate consumer surplus $CS(\mathbf{q})$ (dark green area) and profits $\Pi(\mathbf{q})$ (light green area) between 1994 and 2018 for the observed *Common Ownership* equilibrium. Total surplus W is the combined area of $CS(\mathbf{q})$ and $\Pi(\mathbf{q})$. On the right axis we also plot profits as a share of total surplus Π/W (dotted black line).

The total surplus produced by U.S. public corporations almost tripled between 1994 and 2018 from \$3.116 trillion to \$8.953 trillion. Most of the increase over this time period is due to the increase in profits while the gains in consumer surplus have been comparatively modest. Profits increased from \$1.597 trillion to \$5.639 trillion. Consumer surplus increased from \$1.519 trillion in 1994 to \$3.314 trillion in 2018. Because of these opposing shifts, the profit share increased from 51.2% of total surplus in 1994 to 63% in 2018, but consumer surplus dropped from 48.8% of total surplus to 37% in the same time period.

To investigate the evolution of the profit share in greater detail and to decompose the separate

[16] Because our model uses both HP's time-varying product similarity data and time-varying ownership, our estimates account for how the product offering of U.S. public firms and their ownership has changed over time.

FIGURE 6: DEADWEIGHT LOSS

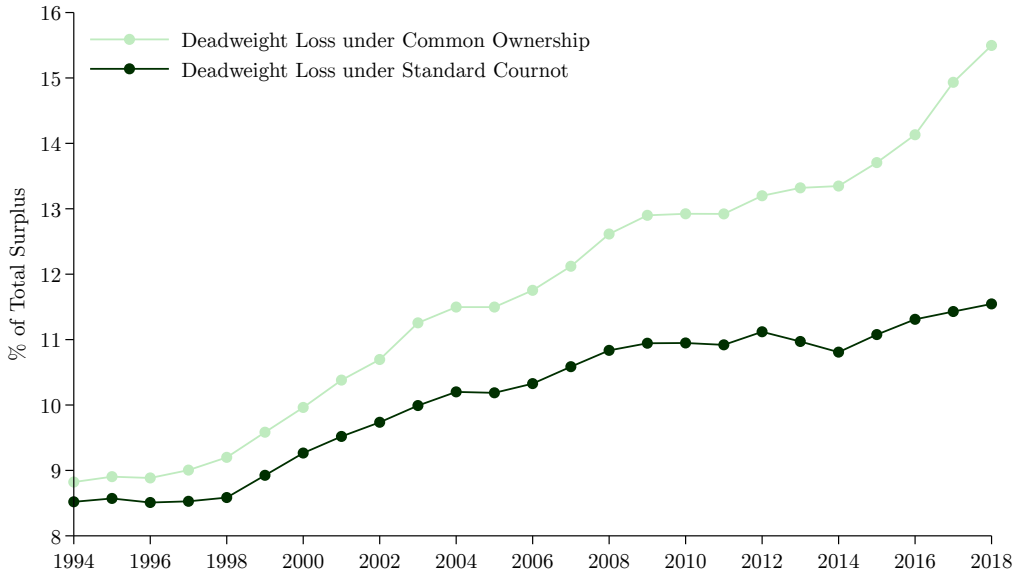

FIGURE NOTES: The figure plots the estimated deadweight loss (DWL) of oligopoly and of oligopoly and common ownership, between 1994 and 2018. The dark green line is the DWL of oligopoly, the % difference in total surplus between the Cournot equilibrium and the Perfect Competition scenario. The light green line is the % difference between the Cournot Common Ownership equilibrium and the Perfect Competition scenario.

effects of oligopoly and common ownership we plot the profit share of total surplus under Cournot with and without common ownership in Figure 5. Under standard Cournot without common ownership (dark green line) the increase in the profit share is significantly less pronounced than under Cournot with common ownership (light green line). Under standard Cournot the profit share only increases by 5.5 percentage points from 50.6% to 56.1%. In contrast, the increase in the profit share under common ownership is almost twice as large. The profit share increases by almost 12 percentage points from 51.2% to 63%.

Figure 6 plots the respective percentage gains in total surplus when the economy moves from the standard Cournot equilibrium $\mathbf{q}^{\Psi}$ and from the CCO equilibrium $\mathbf{q}^{\Xi}$ to the first-best perfect competition equilibrium $\mathbf{q}^{W}$. These are the deadweight losses of oligopoly (dark green line) and of the combination of oligopoly and common ownership (light green line). Their respective trends closely mimic those of the profit shares of total surplus under both of these regimes. The deadweight losses increase from 8.5% and 8.8% in 1994 to 11.5% and 15.5% in 2018 suggesting increasingly harmful effects of oligopolistic behavior and common ownership.

The primary focus of our paper is to consider the welfare impact of common ownership over and above the impact of oligopoly. The left panel of Figure 7 plots the evolution of the deadweight loss that is solely due to the presence of common ownership. Specifically, the figure plots the difference between the two lines in Figure 6. This is the difference between the % difference in total

FIGURE 7: COMMON OWNERSHIP: DEADWEIGHT LOSS AND DISTRIBUTIONAL EFFECTS

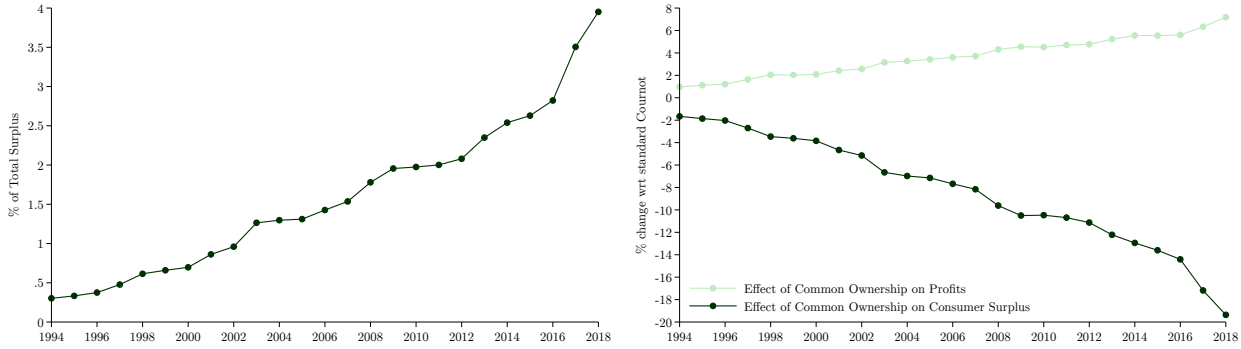

FIGURE NOTES: The left panel of the figure above plots the deadweight loss from common ownership, measured as the difference in total surplus (W) between the Cournot oligopoly allocation and the CCO allocation. The right panel displays the effect of common ownership on profits and consumer surplus, measured as the percentage difference between the Cournot oligopoly allocation and the CCO allocation from 1994 to 2018.

surplus between standard Cournot and perfect competition and the % difference in total surplus between Cournot with common ownership and perfect competition. Whereas the deadweight loss attributable to common ownership is relatively modest in 1994 (0.3% of total surplus), it increases more than tenfold over the course of our sample reaching 4% of total surplus in 2018. As a result, the increase in deadweight loss under Cournot with common ownership (Figure 6, light green line) from 8.8% in 1994 to 15.5% in 2018 is due in slightly larger part to common ownership (59.7% of the increase) than to standard oligopoly reasons (40.3%).

From an antitrust perspective we are particularly interested in the effect of common ownership on consumer surplus and its evolution over time. In the right panel of Figure 7 we plot the effect of common ownership on corporate profits and consumer surplus from 1994 to 2018. Common ownership raised corporate profits by 1% in 1994 and by 6.6% in 2018. At the same time, it lowered consumer surplus by less than 1.7% in 1994 but by almost 20% in 2018.

Taken together, our results suggest that, compared to 1994, U.S. public firms have more market power in 2018 due to both standard oligopolistic reasons as well as due to an increase in ownership concentration and overlap. According to our estimates this increase in aggregate market power negatively impacted both allocative efficiency and consumer welfare.

5 Robustness

5.1 Alternative Corporate Governance Assumptions

We now consider alternative assumptions of corporate governance that lead to different objective functions for the firm. For each of these alternative governance models, we then perform welfare calculations (as we did for the baseline model). This allows us to investigate the sensitivity of our

results to these governance assumptions.

5.1.1 Superproportional Influence of Large Investors

The governance model previously presented assumes that each firm i maximizes the profit shares of its investors, weighting them in proportion to the stake they own, when setting q_i. However, there are good reasons to believe that larger investors exert influence that exceeds the size of their stake. That is to say, proportional influence as assumed by Rotemberg (1984) may over- or understate the importance of large investors for strategic firm decisions. One reason this might be the case is that corporate voting is more akin to majoritarian rather than proportional representation (Azar and Vives, 2021b).

Suppose that firm i weights the profits of investor z by $s_{iz} \cdot \gamma_{iz}$ (as opposed to s_{iz}). Here γ_{iz} is an investor "influence" weight that can increase or decrease the weight assigned to a particular investor depending on how large the investor's stake in company i is. We can then define the following influence-adjusted common ownership weights. $\tilde{\kappa}_{ij}$ by

$$\tilde{\kappa}_{ij} \overset{\text{def}}{=} \frac{\sum_{z=1}^{Z} s_{iz}\gamma_{iz}s_{jz}}{\sum_{z=1}^{Z} s_{iz}\gamma_{iz}s_{jz}} \tag{5.1}$$

The analysis of Gilje et al. (2020) suggests that these influence weights are concave in the ownership shares of investors. Consistent with their approach (and with that of Backus et al. 2021b), we approximate this influence function using a square root (i.e., $\gamma_{iz} \overset{\text{def}}{=} \sqrt{s_{iz}}$).

5.1.2 Blockholder Thresholds

An alternative way to model the fact that large investors exercise a disproportional level of control over a corporation is to introduce "blockholders." These are investors that are presumed to actively exercise control on some company i if their stake in i exceeds a certain threshold.

The literature typically defines a blockholder as a shareholder holding 5% or more of a company's stock, due the fact that this level triggers additional SEC disclosure requirements (Edmans and Holderness, 2017). Blockholders have been shown to play an important role in ensuring that there is at least one owner who has the correct incentives to make residual decisions in a way that creates value. Their influence can come through direct intervention in a firm's operations (otherwise known as "voice") and through selling of shares if the firm underperforms (otherwise known as "exit").

We construct blockholding-consistent common ownership weights, based on the alternative assumption that investors exert influence only when their ownership stake exceeds the 5% blockholder threshold in a company. That is, if some investor z is a blockholder of firm i, the firm is assumed to internalize the impact of its production decision on the investor's portfolio profits. Otherwise, the impact on the investor's portfolio profits is disregarded and the investor's portfolio profits are assumed to coincide with the firm's profit function. Specifically, the manager i's objective function

becomes:

$$\phi_i \quad = \quad \pi_i \sum_{z=1}^{Z} s_{iz} s_{iz} + \sum_{j \neq i} \pi_j \sum_{z=1}^{Z} s_{iz} b_{iz} s_{jz} \tag{5.2}$$

where b_{iz} is a dummy variable that identifies whether investor z is a blockholder of company i. The blockholder-adjusted common ownership weights are thus given by

$$\tilde{\kappa}_{ij} \overset{\text{def}}{=} \frac{\sum_{z=1}^{Z} s_{iz} b_{iz} s_{jz}}{\sum_{z=1}^{Z} s_{iz} s_{jz}} \quad \text{for } i \neq j \tag{5.3}$$

in line with the SEC's definition of a "blockholder." We assume that $b_{iz} = 1$ if and only if $s_{iz} > 5\%$.

5.1.3 Rational Investor Inattention

One of the assumptions of the governance model previously presented is that each firm i fully internalizes the weighted profit shares of its investors when choosing q_i. While intuitively appealing, this assumption may not be entirely realistic. Agency problems between owners and managers may attenuate or even exacerbate the anticompetitive effects of common ownership (Antón et al., 2020). Similarly, (Gilje, Gormley and Levit, 2020, henceforth GGL) have highlighted the importance of investor inattention in evaluating the effects of common ownership. Investor attention here refers to the extent to which firm owners incorporate strategic considerations related to common ownership in influencing a company's decision. The rationale is that monitoring a firm's management and forcing it to incorporate strategic considerations related to common ownership requires a cost from the investor. Incurring this cost might not be optimal for every investor. This is likely to be the case for firm holdings that constitute only a small portion of a large, diversified investor's overall portfolio.

Motivated by this consideration, GGL propose a corporate governance model of common ownership, which produces the following alternative measure of firm i's sympathy towards firm j:

$$\text{GGL}_{ij}^{\text{full}} \quad \overset{\text{def}}{=} \quad \mathbf{s}_i' \mathbf{s}_j \quad \equiv \quad \sum_{z=1}^{Z} s_{iz} s_{jz} \tag{5.4}$$

While GGL's sympathy score differs from the welfare-relevant measure of common ownership in our structural model (κ_{ij}), the two metrics are closely related. It can be immediately verified that

$$\kappa_{ij} \quad = \quad \frac{\text{GGL}_{ij}^{\text{full}}}{\text{IHHI}_i} \tag{5.5}$$

where IHHI_i is the investor Herfindahl index of firm i. This specific measure of common ownership, just like κ_{ij}, presumes that investors are fully attentive to the product market interactions of the firms in their portfolio. GGL generalize their measure to a setting where investors are allowed to

be imperfectly attentive:

$$\text{GGL}_{ij}^{\text{fitted}} \quad \stackrel{\text{def}}{=} \quad \sum_{z=1}^{Z} s_{iz}\, g_{iz}\, s_{jz} \tag{5.6}$$

where $g_{iz} \in [0,1]$ is an "attention weight," which captures the degree to which investor z internalizes the product market rivalries of firm i. Following Iliev and Lowry (2015)'s approach of using proxy voting data, GGL estimate these attention weights non-parametrically as the probability of investor z deviating from the voting recommendation issued by the Institutional Shareholders Service (ISS), conditional on the weight of firm i in investor z's portfolio. They find that the higher the weight of firm i in z's portfolio, the higher the likelihood of z deviating from ISS's recommendation. The intuition behind this measure is that an inattentive investor will completely delegate their voting choices to ISS and never disagree with their recommendation. The assumption is that ISS itself does not base their voting recommendations on product market rivalry considerations.

We can use the attention weights of GGL to compute a modified κ_{ij} that accounts for imperfect investor attention. To do so, however, we need to make an assumption about the ex-ante probability that a fully-attentive investor disagrees with ISS. Because the disagreement probabilities estimated by GGL top out at 7% (i.e., the probability in the limit case in which a company comprises the entirety of an investor's portfolio), we assume that 7% is the disagreement probability in the full attention case. This assumption is consistent with investors paying full attention to companies that make up the entirety of their portfolio.

This leads to the following inattention-modified sympathy weight $\bar{\kappa}_{ij}$ given by

$$\bar{\kappa}_{ij} \quad \stackrel{\text{def}}{=} \quad \frac{1}{0.07} \cdot \frac{\text{GGL}_{ij}^{\text{fitted}}}{\text{IHHI}_i} \quad = \quad \frac{1}{0.07} \cdot \underbrace{\frac{\text{GGL}_{ij}^{\text{fitted}}}{\text{GGL}_{ij}^{\text{full}}}}_{<0.07} \cdot \kappa_{ij} \tag{5.7}$$

which can be computed using the data of GGL which is available on the WRDS platform.

This measure can be micro-founded with a behavioral corporate governance model in which firm i's management discounts investor z's share of the competitors' profits at a rate g_{iz}, which is consistent with the following objective function for firm i:

$$\phi_i \quad = \quad \pi_i \sum_{z=1}^{Z} s_{iz} s_{iz} + \sum_{j \neq i} \pi_j \sum_{z=1}^{Z} s_{iz} g_{iz} s_{jz} \quad \text{for } i \neq j \tag{5.8}$$

Note that, by construction, $0 \leq \bar{\kappa}_{ij} \leq \kappa_{ij}$. Therefore, in the framework of GGL, investor inattention dampens the product market effects of common ownership.

5.1.4 Passive Investors, Not Passive Owners

One frequent criticism of the common ownership literature is that because many of the largest common owners are passive investors (i.e., investors who follow a passive investing strategy), they

FIGURE 8: COMMON OWNERSHIP DWL - ALTERNATIVE GOVERNANCE MODELS

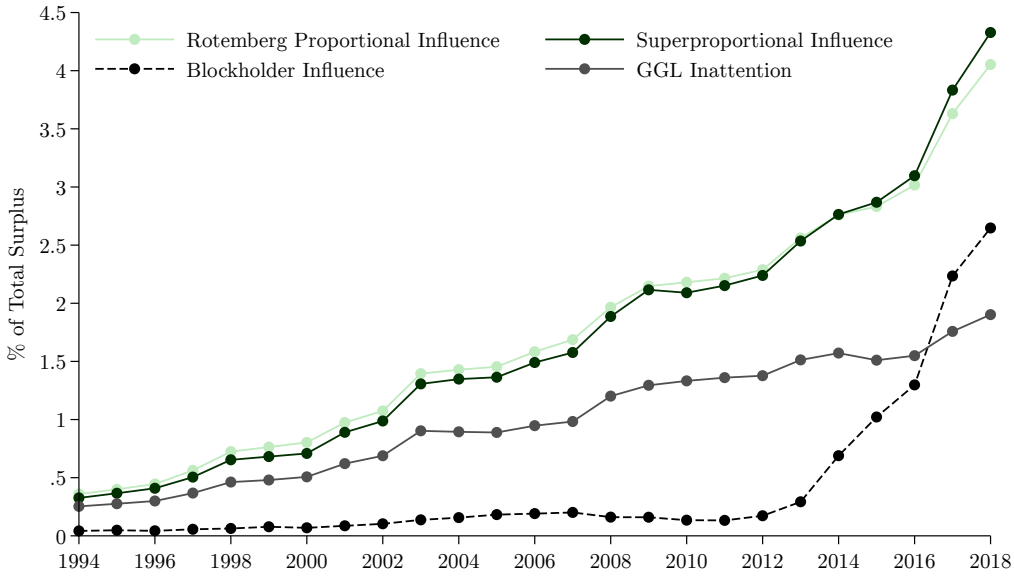

FIGURE NOTES: The figure plots the deadweight loss of common ownership, computed as % of the total surplus, under proportional, superproportional and blockholder influence models, and the GGL inattention model from 1994 to 2018.

cannot and do not affect firm decisions. Appel, Gormley and Keim (2016) empirically investigate the common misconception that such passive investors are by definition passive owners (i.e., owners who do not influence firms' governance). In fact, their analysis suggests the opposite: passive mutual funds play a rather active role in firms' governance choices and thus passive investing does not necessarily equate with passive ownership.[17] Therefore, in our analysis we do not specifically distinguish between active and passive investors.

5.1.5 Empirical Results

We now compare the results of these alternative governance assumptions to our benchmark case which assumes Rotemberg (i.e., proportional) common ownership weights.

In Figure 8 we plot the evolution of the deadweight loss that is due to the presence of common ownership under different governance assumptions. Whereas superproportional influence of large investors leads to a deadweight loss that is quite similar though slightly larger than under proportional common ownership throughout our sample, the effect of common ownership with blockholder thresholds is much smaller in the early years of our sample. Until 2013 the deadweight loss of blockholder common ownership is well below 0.5% of total surplus. However, after that it rises rapidly to as high as 2.5% of total surplus at the end of our sample period. This is in large part due to the

[17] Furthermore, as Antón et al. (2020) show both theoretically and empirically, the relative governance passivity of common owners does not imply that there are no anticompetitive product market effects of common ownership.

FIGURE 9: DISTRIBUTIONAL EFFECTS - ALTERNATE GOVERNANCE MODELS (1994-2018)

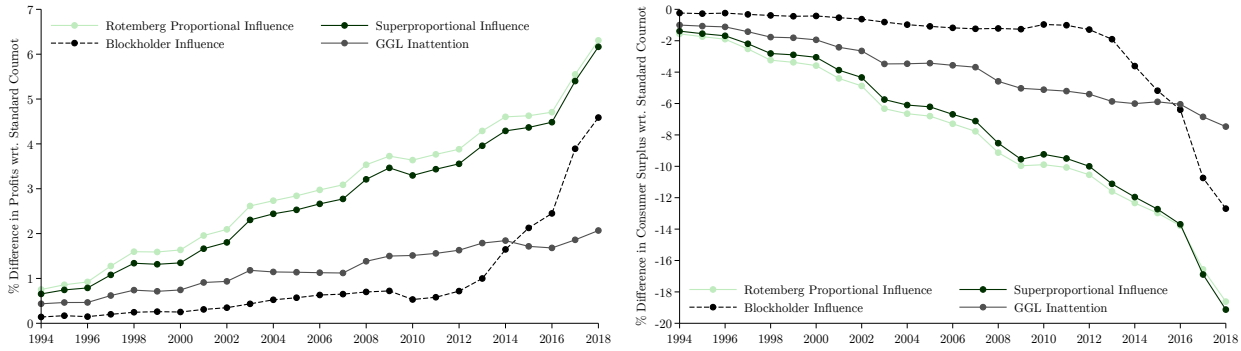

FIGURE NOTES: The figure shows the effect of common ownership on profits (left panel) and consumer surplus (right panel), measured as the percentage difference between the Cournot and the CCO allocation from 1994 to 2018.

increasingly large ownership stakes of the biggest asset management companies in all publicly listed firms. Until the mid-2010s their ownership stakes rarely exceeded the 5% blockholder threshold, but by the end of the sample they constitute the top shareholders for almost all publicly listed firms. For example, today both BlackRock and Vanguard are among the top five shareholders of almost 70 percent of the largest 2,000 publicly traded firms in the U.S. whereas twenty years ago that number was zero percent for both firms (Fichtner, Heemskerk and Garcia-Bernardo, 2017). Finally, the deadweight loss estimates of common ownership under the GGL inattention model hover between the blockholder and proportional influence models until 2014. Even under GGL inattention, common ownership leads to a deadweight loss of almost 2% of total surplus, much higher than in 1994. One caveat of our empirical implementation of the GGL inattention model is that GGL's data are only available up to 2012. For the years following 2012 we have to use the 2012 attention weights. This means that while the modified sympathy weights $\bar{\kappa}_{ij}$ still capture the increase in ownership concentration that takes place in this period, they fail to reflect changes in (in)attention that may have taken place at the same time.

In Figure 9 similar patterns emerge for the distributional consequences of common ownership on firm profits (left panel) and consumer surplus (right panel). Common ownership with superproportional influence leads to essentially identical increases in profits and decreases in consumer surplus as our benchmark case with Rotemberg proportional weights. Common ownership with blockholder influence thresholds have little impact on either measure until about 2012. However, even with blockholder thresholds common ownership raises firm profits by almost 5% of total surplus and lowers consumer surplus by almost 13% in 2018. Under GGL governance assumptions common ownership has more muted distributional consequences. Common ownership modestly raises firm profits by approximately 2%, but still leads to a significant reduction of consumer surplus of almost 7% in 2018.

Thus, even under alternative governance assumptions common ownership leads to a sizeable

deadweight loss that is increasing over time as well as considerable distributional consequences that transfer rents from consumers to producers.

5.2 Fixed Costs and Intangible Capital

Thus far, our analysis abstracted from fixed costs. We now explore whether accounting for fixed costs in the firms' production function affects our estimates. To include fixed costs we modify firm i's total cost function as follows:

$$h_i \;=\; f_i + c_i q_i + \frac{\delta_i}{2} q_i^2 \tag{5.9}$$

where f_i is the fixed cost component which we measure using the Compustat variable "Selling, General and Administrative Costs" (SG&A).

We also consider whether higher profits (defined as the entire capital compensation) are the result of a higher required return on capital that is not explicitly modeled. For this reason, in addition to fixed costs, we also subtract from firm profits (π_i) a measure of the minimum required profits ($\hat{\pi}_i$). These minimum required profits are assumed to be an exogenous variable which we measure as the product of the total capital stock and the user cost of capital. The latter is computed using the methodology of De Loecker et al. (2020) and Pellegrino (2019).

The computation of f_i and $\hat{\pi}_i$ is, however, affected by the treatment of SG&A. In Compustat, this item contains miscellaneous costs that include R&D expenditures and are not directly linked to production. SG&A also partly includes investments in *intangible capital*. As a result, some authors argue that it should not be treated as overhead but should instead be capitalized. If we choose to *not* capitalize (i.e., expense) intangible investments, we simply measure fixed costs using SG&A and the capital stock using property plant and equipment (PPE):

$$f_i^{\text{exp}} \;=\; \text{SG\&A}_i \qquad \text{and} \qquad \hat{\pi}_i^{\text{exp}} \;=\; \text{PPE}_i \times \text{User Cost of Capital} \tag{5.10}$$

We then study how the evolution of the deadweight loss and of the profit share changes when we subtract all the fixed costs ($F \stackrel{\text{def}}{=} \sum_i f_i$) as well as all the minimum required profits ($\hat{\Pi} \stackrel{\text{def}}{=} \sum_i \hat{\pi}_i$) from Π and W. Does the profit share of total surplus still increase?

In Figures 10 and 11 in the appendix we show the evolution of the profit share and the deadweight loss after redefining Π and W to be computed net of fixed costs F and minimum required profits $\hat{\Pi}$. In contrast to Figure 5 and 6, when fixed costs and minimum required profits are subtracted the profit share of total surplus in Figure 10 increases more dramatically because it starts from a lower level in 1994.

How do these results change if we instead capitalize intangible investments? Peters and Taylor (2017) capitalize intangible investment by treating R&D expenditures and 30% of the remaining portion of SG&A as investment in intangible capital. This approach leads to the following alterna-

tive measures of fixed costs and minimum required profits:

$$f_i^{\text{cap}} = (\text{SG\&A}_i - \text{R\&D}_i) \times 0.7 \qquad \text{and} \qquad \hat{\pi}_i^{\text{cap}} = (\text{PPE}_i + \text{Intan. Capital}_i) \times \text{User CoC} \quad (5.11)$$

Figures 12 and 13 use this alternative measure of fixed costs in which intangible investments are capitalized rather than expensed. Our results are largely unaffected by using these alternative measures of fixed costs and minimum profits.

5.3 Private and Foreign Firms

As in Backus et al. (2021b), our empirical analysis focuses on publicly listed U.S. firms and does not include foreign and privately held firms because we ownership data for these firms is not as readily available. To the extent that these firms do not share much overlap in ownership with the publicly listed U.S. firms that are in our sample, our results on the welfare costs and distributional consequences of common ownership are robust to this challenge.

Nonetheless, to circumvent these data problems our model can be extended to include a continuum of atomistic firms that we assume to behave competitively, to share the same degree of ownership, and whose market activity is determined by a productivity cut-off value (Hopenhayn, 1992; Pellegrino, 2019). An extended analysis which includes this competitive fringe of (private and foreign) firms is available on request.

5.4 Bertrand Oligopoly

In this paper, we deliberately focus on Cournot competition because we obtain a model that is both tractable and flexible. All of our analyses can be replicated under the assumption of Bertrand competition. However, the Bertrand case is significantly less tractable, more computationally involved, and the resulting equilibrium equations are harder to interpret. In Appendix D we derive the Bertrand equilibrium with common ownership for the case with a flat marginal cost function ($\boldsymbol{\Delta} = \mathbf{0}$).

Nonetheless, it is interesting to consider how our results would change if we assumed price as opposed to quantity competition. Because Bertrand competition in our model is more intense than Cournot competition and hence results in higher equilibrium quantities, the deadweight loss from oligopoly is smaller when firms set prices rather than quantities. However, because the monopoly solution is independent of whether prices or quantities are chosen and common ownership moves the economy closer to the monopoly potential, the incremental anticompetitive effect of common ownership relative to the standard oligopoly solution is more pronounced under Bertrand than under Cournot competition. Hence, our estimates of the deadweight loss of common ownership obtained under Cournot competition are more conservative.

6 Conclusions

In this paper we provide the first quantification of the welfare and distributional effects of common ownership at the macroeconomic rather than just the industry level. We develop a general equilibrium model of oligopoly in which firms are connected through two large networks of product similarity and ownership overlap. Our baseline empirical estimates indicate that the rise of common ownership from 1994 to 2018 led to considerable and increasing deadweight losses, amounting to 0.3% of the total surplus in 1994 and as much as 4% by 2018. In addition, the increase in common ownership resulted in a significantly lower share of total surplus accruing to consumers. The key insights of our findings also continue to hold for alternative corporate governance assumptions such as superproportional influence, blockownership thresholds, and limited investor attention. The economically large impact of common ownership in several industries across the entire economy as well as its continuing increase suggest that antitrust policy and financial regulation may have to address this new challenge.

In our analysis we assume a particular form of firm conduct (i.e., quantity choices in Cournot product market oligopoly) and therefore focus on a particular set of welfare implications of common ownership (i.e., static unilateral effects on product market rivalry). However, there are several other firm decisions which are affected by common ownership. For example, recent theoretical and empirical contributions suggest that common ownership may also affect labor market power (Azar and Vives, 2021a), innovation (Antón et al., 2018; López and Vives, 2019; Eldar et al., 2020; Li et al., 2020), entry (Newham et al., 2019; Xie and Gerakos, 2020), firm productivity and cost efficiency (Antón et al., 2020), and incentives to collude (Pawliczek, Skinner and Zechman, 2019; Shekita, 2021). The potential quantification of these additional channels through which common ownership ultimately affects outcomes across the entire economy, provides a number of fruitful research opportunities to which our flexible theoretical framework and empirical methodology can be applied in the future.

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Appendices

A Tale of Two Networks:
Common Ownership and Product Market Rivalry

Florian Ederer (Yale) & Bruno Pellegrino (UMD)

A Correction for Unobserved Ownership

In this appendix, we detail our methodology to estimate the matrix of profit weights $\mathbf{K}$, in a way that is robust to the presence of unobserved investors which would otherwise lead to a thick right tail of implausibly large κ_{ij}. We start by rewriting κ_{ij} in the following way:

$$\kappa_{ij} \;=\; \frac{\mathbf{s}_i'\mathbf{s}_j}{\text{IHHI}_i} \tag{A.1}$$

The key problem is to estimate the numerator and the denominator based on the fact that in 13(f) data we observe a limited set of investors. Let us denote with O the set of Observed Investors, and $\mathcal{U}$ the set of Unobserved Investors.

Importantly, the denominator of the of vector $\mathbf{s}_i$, which is the total number of shares, includes both observed and unobserved investors because it is taken from Compustat. Hence, typically, the observed s_{iz} will sum to a value less than one.

All the diagonal κ_{ii} are equal to one by construction and hence we can focus on the $i \neq j$ case. Under the (conservative) assumption that there is zero overlap in ownership between i and j among unobserved investors:

$$\sum_{z\in\mathcal{U}} s_{iz}s_{jz} = 0 \tag{A.2}$$

we can compute the numerator of the equation above by simply ignoring the unobserved investors.

Estimating the denominator is slightly more complex. If we compute the IHHI using observed investors only we obtain:

$$\widehat{\text{IHHI}}_i \;=\; \sum_{k\in\mathcal{O}} s_{iz}^2 \tag{A.3}$$

a downward biased estimate of the IHHI. For some firms, where few small investors are observed, this bias can be enormous, leading κ to exceed 10,000. Let us write the "true" IHHI as

$$\text{IHHI}_i^* \;=\; \sum_{z\in\mathcal{O}} s_{iz}^2 + \sum_{z\in\mathcal{U}} s_{iz}^2 \tag{A.4}$$

Let $S_{i(\mathcal{O})}$ and $S_{i(\mathcal{U})}$ be the sum of shares for the observed and unobserved investors, respectively:

$$S_{i(\mathcal{O})} = \sum_{z \in \mathcal{O}} s_{iz} \qquad S_{i(\mathcal{U})} = \sum_{z \in \mathcal{U}} s_{iz} \tag{A.5}$$

and let $s_{i(\mathcal{O})k}$ and $s_{i(\mathcal{U})k}$ the shares owned by investor k as a share of the observed and unobserved ones, respectively:

$$s_{i(\mathcal{O})z} = \frac{1}{S_{i(\mathcal{O})}} \cdot \sum_{z \in \mathcal{O}} s_{iz} \qquad s_{i(\mathcal{U})z} = \frac{1}{S_{i(\mathcal{U})}} \cdot \sum_{z \in \mathcal{U}} s_{iz} \tag{A.6}$$

As a result we have

$$\begin{aligned} \text{IHHI}_i^* &= \sum_{z \in \mathcal{O}} \left(S_{i(\mathcal{O})} \cdot s_{i(\mathcal{O})k}\right)^2 + \sum_{z \in \mathcal{U}} \left(S_{i(\mathcal{U})} \cdot s_{i(\mathcal{U})k}\right)^2 \\ &= S_{i(\mathcal{O})}^2 \cdot \sum_{z \in \mathcal{O}} s_{i(\mathcal{O})k}^2 + S_{i(\mathcal{U})}^2 \cdot \sum_{z \in \mathcal{U}} s_{i(\mathcal{U})z}^2 \\ &= S_{i(\mathcal{O})}^2 \cdot \text{IHHI}_i^{\mathcal{O}} + S_{i(\mathcal{U})}^2 \cdot \text{IHHI}_i^{\mathcal{U}} \end{aligned}$$

where we have rewritten the terms in summation as the Herfindahl index among *observed* and *unobserved* investors only, respectively. By making the assumption that ownership concentration is identical among unobserved and observed investors $\left(\text{IHHI}_i^{\mathcal{O}} = \text{IHHI}_i^{\mathcal{U}}\right)$, and using the fact that

$$S_{i(\mathcal{U})} = 1 - S_{i(\mathcal{O})} \tag{A.7}$$

the true Herfindahl index can be rewritten as:

$$\begin{aligned} \text{IHHI}_i^* &= \left[S_{i(\mathcal{O})}^2 + \left(1 - S_{i(\mathcal{O})}\right)^2\right] \cdot \text{IHHI}_i^{\mathcal{O}} \\ &= \left[S_{i(\mathcal{O})}^2 + \left(1 - S_{i(\mathcal{O})}\right)^2\right] \cdot \sum_{i \in \mathcal{O}} \left(\frac{1}{S_{i(\mathcal{O})}} s_i\right)^2 \\ &= \left[1 + \left(\frac{1 - S_{i(\mathcal{O})}}{S_{i(\mathcal{O})}}\right)^2\right] \cdot \widehat{\text{IHHI}}_i \end{aligned}$$

The term in square brackets is the correction that we apply to our estimates of the denominator of κ_{ij}.

The fact that we correct the numerator upward, while not correcting the numerator $(\mathbf{s}_i'\mathbf{s}_j)$ implies that our estimate of κ_{ij} provides a lower bound, because including the unobserved investors in the summation can only increase the value of the numerator. This in turn implies that our estimates of the welfare impact of common ownership are, by construction, conservative.

While we think that a more conservative estimate is preferable, there are also downsides. If 13(f) data coverage of institutional shareholdings improved or worsened over time (which is possible but hard to verify), there is a possibility that the downward bias in the numerator (which we chose

to tolerate in order to be conservative) might have become smaller or larger over time. This could potentially bias the trend of our estimates.

In other words, if we chose to correct *both* the numerator and the denominator, we would (mechanically) obtain a larger welfare impact from common ownership but also potentially a different trend. A similar derivation as the one we used for the denumerator leads to the following correction for the numerator:

$$\left(\mathbf{s}_i'\mathbf{s}_j\right)^* \;=\; \left[1+\frac{\left(1-S_i\right)\left(1-S_j\right)}{S_iS_j}\right]\left(\mathbf{s}_i'\mathbf{s}_j\right)^{\mathcal{O}} \tag{A.8}$$

An additional robustness check where we apply this additional correction to the numerator of the common ownership weight is available upon request.

B Welfare Estimates, Profit Impact, and Fixed Costs

In this appendix, we provide welfare estimates for the beginning of our sample as well as for the end of our sample when all κ_{ij} are restricted to be smaller than 1. We also document that common ownership differentially affects corporate profits. Finally, we recompute our estimates of the evolution of the profit share and the deadweight loss over time using different definitions of aggregate profits that subtract fixed costs and minimum required profits.

TABLE 3: WELFARE ESTIMATES (1994)

	Scenario	*Common Ownership*	*Cournot-Nash*	*Perfect Competition*	*Monopoly*
		(1)	(2)	(3)	(4)
Welfare Statistic	**Variable**	$\mathbf{q}^{\Xi}$	$\mathbf{q}^{\Psi}$	$\mathbf{q}^{W}$	$\mathbf{q}^{\Pi}$
Total Surplus (US$ trillions)	$W(\mathbf{q})$	3.116	3.126	3.417	2.755
Aggregate Profits (US$ trillions)	$\Pi(\mathbf{q})$	1.597	1.581	0.723	1.919
Consumer Surplus (US$ trillions)	$\mathrm{CS}(\mathbf{q})$	1.519	1.545	2.694	0.836
Total Surplus / Perfect Competition	$\frac{W(\mathbf{q})}{W(\mathbf{q}^{W})}$	0.912	0.915	1.000	0.806
Aggregate Profit / Total Surplus	$\frac{\Pi(\mathbf{q})}{W(\mathbf{q})}$	0.512	0.506	0.212	0.697
Consumer Surplus / Total Surplus	$\frac{\mathrm{CS}(\mathbf{q})}{W(\mathbf{q})}$	0.488	0.494	0.788	0.303

TABLE NOTES: The table reports the model estimates of aggregate profits, consumer surplus, and total surplus for each of the counterfactual scenarios presented in Section 2.

Table 4: Difference between CCO and Cournot Profits for Selected Companies

Top 10 Companies ranked by $ profit difference (in millions), 2018

Company Name	**CCO Profits**	**Cournot Profits**	**Difference**	**% Diff.**
Wells Fargo & Co	$ 84,543.8	$ 80,947.0	+$ 3,596.7	+4.4%
JPMorgan Chase & Co	$ 104,375.4	$ 100,902.6	+$ 3,472.8	+3.4%
Bank of America	$ 88,779.1	$ 85,979.4	+$ 2,799.6	+3.3%
Verizon Communications	$ 75,700.7	$ 73,000.2	+$ 2,700.5	+3.7%
Walmart	$ 133,912.6	$ 131,642.8	+$ 2,269.8	+1.7%
Microsoft	$ 81,894.8	$ 79,643.0	+$ 2,251.8	+2.8%
Alphabet	$ 86,288.4	$ 84,378.8	+$ 1,909.6	+2.3%
AT&T	$ 91,397.4	$ 89,514.2	+$ 1,883.2	+2.1%
Apple	$ 112,506.4	$ 110,657.7	+$ 1,848.7	+1.7%
Goldman Sachs Group	$ 26,997.7	$ 25,227.9	+$ 1,769.8	+7.0%

Bottom 10 Companies ranked by $ profit difference (in millions), 2018

Company Name	**CCO Profits**	**Cournot Profits**	**Difference**	**% Diff.**
Regenxbio	$ 113.2	$ 136.2	-$ 23.0	-16.9%
Natural Gas Services Group	$ 18.5	$ 42.4	-$ 23.8	-56.2%
Limelight Networks	$ 118.5	$ 143.7	-$ 25.1	-17.5%
Liveperson	$ 152.3	$ 178.5	-$ 26.2	-14.7%
Enterprise Financial Services	$ 160.8	$ 190.8	-$ 30.0	-15.7%
Boingo Wireless	$ 235.0	$ 272.6	-$ 37.7	-13.8%
Triple-S Management	$ 60.9	$ 101.0	-$ 40.0	-39.6%
Tilly's	$ 210.9	$ 252.4	-$ 41.5	-16.4%
Pnm Resources	$ 312.9	$ 355.1	-$ 42.2	-11.9%
Callon Petroleum	$ 375.3	$ 477.6	-$ 102.3	-21.4%

TABLE 5: WELFARE ESTIMATES (2018) FOR $\kappa_{ij} \leq 1$

	Scenario	*Common Ownership*	*Cournot-Nash*	*Perfect Competition*	*Monopoly*
		(1)	(2)	(3)	(4)
Welfare Statistic	**Variable**	$\mathbf{q}^{\Xi}$	$\mathbf{q}^{\Psi}$	$\mathbf{q}^{W}$	$\mathbf{q}^{\Pi}$
Total Surplus (US$ trillions)	$W(\mathbf{q})$	8.406	8.752	9.881	7.922
Aggregate Profits (US$ trillions)	$\Pi(\mathbf{q})$	5.234	4.923	1.935	5.493
Consumer Surplus (US$ trillions)	$S(\mathbf{q})$	3.172	3.830	7.946	2.429
Total Surplus / Perfect Competition	$\frac{W(\mathbf{q})}{W(\mathbf{q}^{W})}$	0.851	0.886	1.000	0.802
Aggregate Profit / Total Surplus	$\frac{\Pi(\mathbf{q})}{W(\mathbf{q})}$	0.623	0.562	0.196	0.693
Consumer Surplus / Total Surplus	$\frac{\text{CS}(\mathbf{q})}{W(\mathbf{q})}$	0.377	0.438	0.804	0.307

TABLE NOTES: The table reports the model estimates of aggregate profits, consumer surplus, and total surplus for each of the counterfactual scenarios presented in Section 2 restricting $\kappa_{ij} \leq 1$.

FIGURE 10: PROFIT SHARE, NET OF FIXED COSTS AND MINIMUM PROFITS

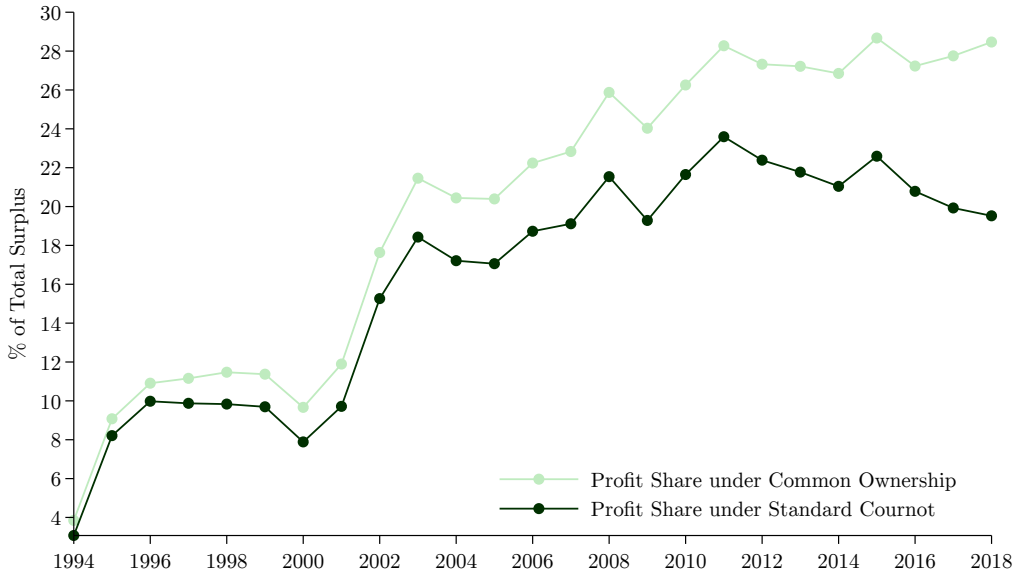

FIGURE NOTES: The figure plots the profit share, net of fixed costs and minimum profits, under standard Cournot (dark green line) and Cournot with common ownership (light green line) between 1994 and 2018.

FIGURE 11: DEADWEIGHT LOSS, NET OF FIXED COSTS AND MINIMUM PROFITS

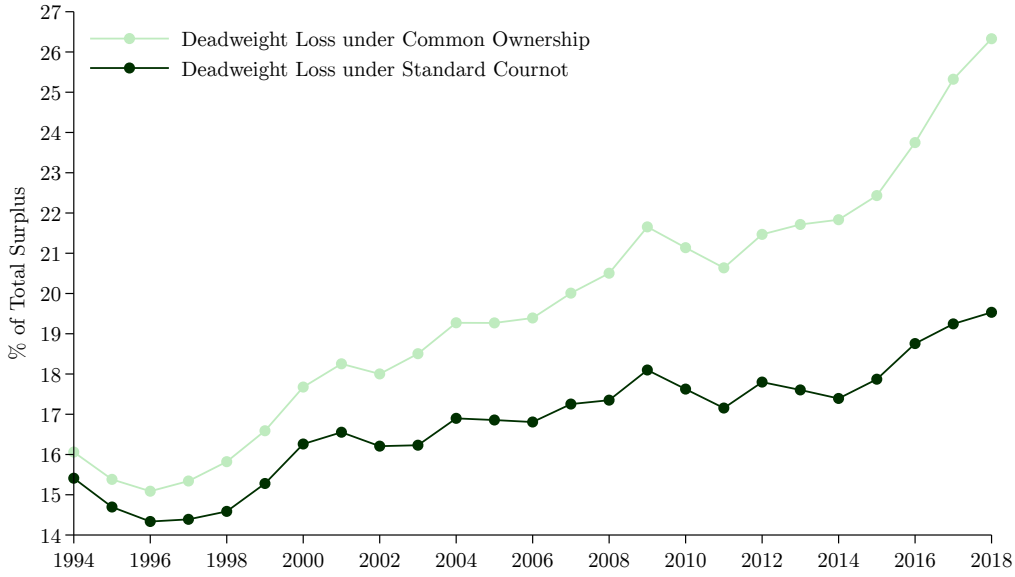

FIGURE NOTES: The figure plots the estimated deadweight loss (DWL), net of fixed costs and minimum profits, of oligopoly and of oligopoly and common ownership between 1994 and 2018. The dark green line is the DWL of oligopoly as measured by the % difference in total surplus between the Cournot equilibrium and the Perfect Competition scenario. The light green line is the DWL of oligopoly and common ownership as measured by the % difference between the Cournot Common Ownership equilibrium and the Perfect Competition scenario.

Figure 12: Profit Share, Net of Fixed Costs and Minimum Profits (Alternative)

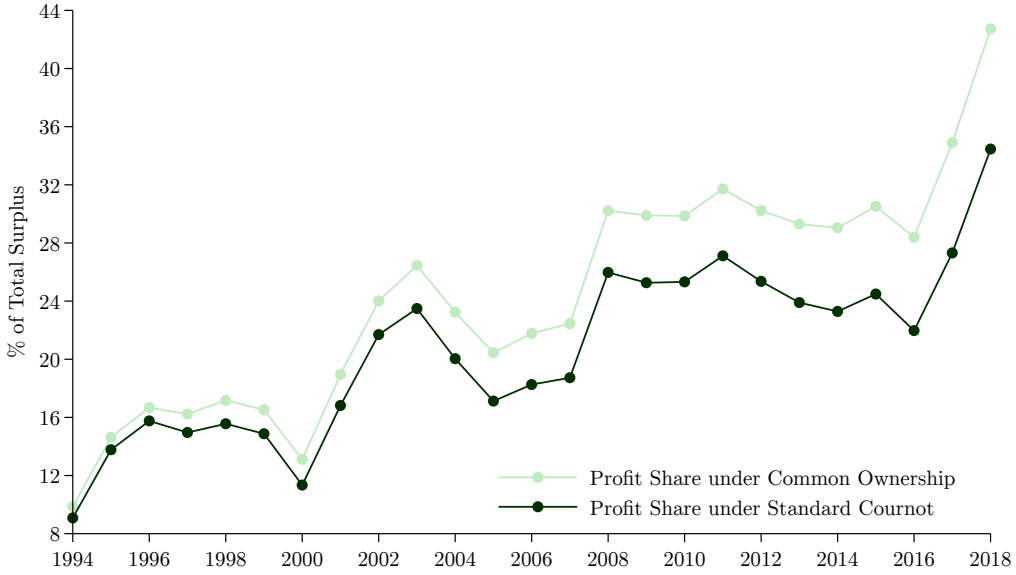

Figure Notes: The figure plots the profit share, net of fixed costs and minimum profits, under standard Cournot (dark green line) and Cournot with common ownership (light green line) between 1994 and 2018. Intangible investments are capitalized rather than expensed.

Figure 13: Deadweight Loss, Net of Fixed Costs and Minimum Profits (Alternative)

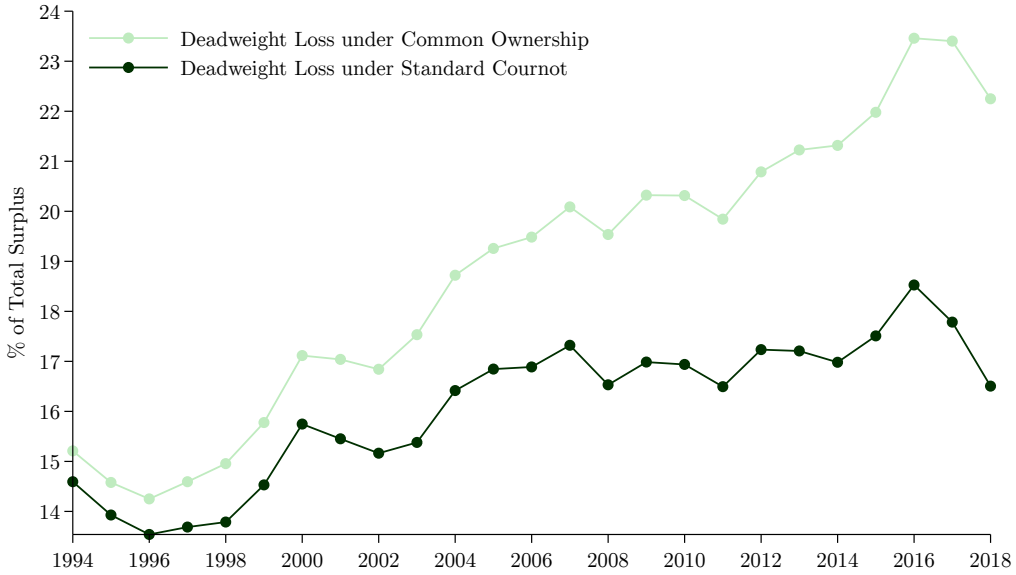

Figure Notes: The figure plots the estimated deadweight loss (DWL), net of fixed costs and minimum profits, of oligopoly and of oligopoly and common ownership between 1994 and 2018. The dark green line is the DWL of oligopoly as measured by the % difference in total surplus between the Cournot equilibrium and the Perfect Competition scenario. The light green line is the DWL of oligopoly and common ownership as measured by the % difference between the Cournot Common Ownership equilibrium and the Perfect Competition scenario. Intangible investments are capitalized rather than expensed.

FIGURE 14: REVENUE-WEIGHTED AVERAGE MARKUP

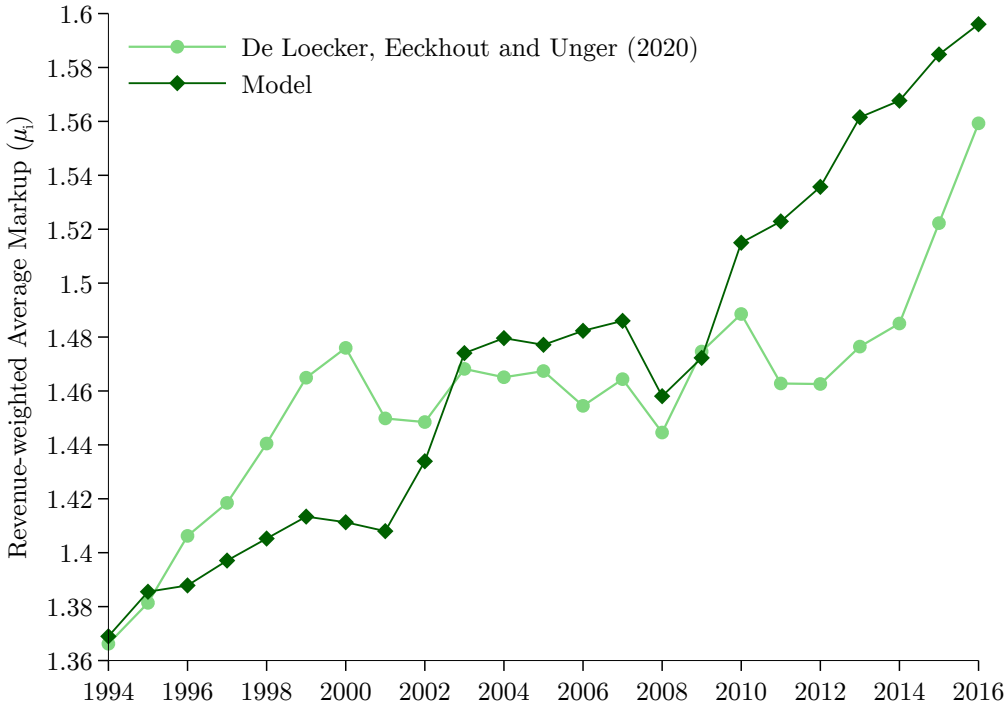

FIGURE NOTES: The figure plots the time series of average markups estimated by our model (dark green line) and by DEU (light green line).

C Calibration

The welfare impact and distributional consequences of oligopoly and common ownership in our model are primarily determined by the markups charged by firms. Thus, to correctly estimate these effects it is imperative that our model generates realistic time-series and cross-sectional variation in markups. In this section, we demonstrate that when combined with data on product similarity and ownership our model does indeed match existing markup estimates across time and across firms from economy-wide studies. In addition, we show that our model-implied markups and cross-price demand elasticities correlate with existing econometric estimates from a host of industry studies.

C.1 Time-series Variation in Markups

To show that it is appropriate to apply Pellegrino (2019)'s α, $\tilde{\delta}$, and θ to our model (despite the change in conduct assumption), we compute markups and cross-price elasticities based on these calibrated values and compare them to empirical values.

Pellegrino (2019) shows that firm i's markup can be written as a function of the firm's capital k_i, revenues π_i, total cost h_i, and the parameter $\tilde{\delta}$. He then calibrates $\tilde{\delta}$ by targeting the average revenue-weighted markup over 1957-1982 and shows that the implied average markups closely tracks

FIGURE 15: CALIBRATED MARKUPS VS. DEU

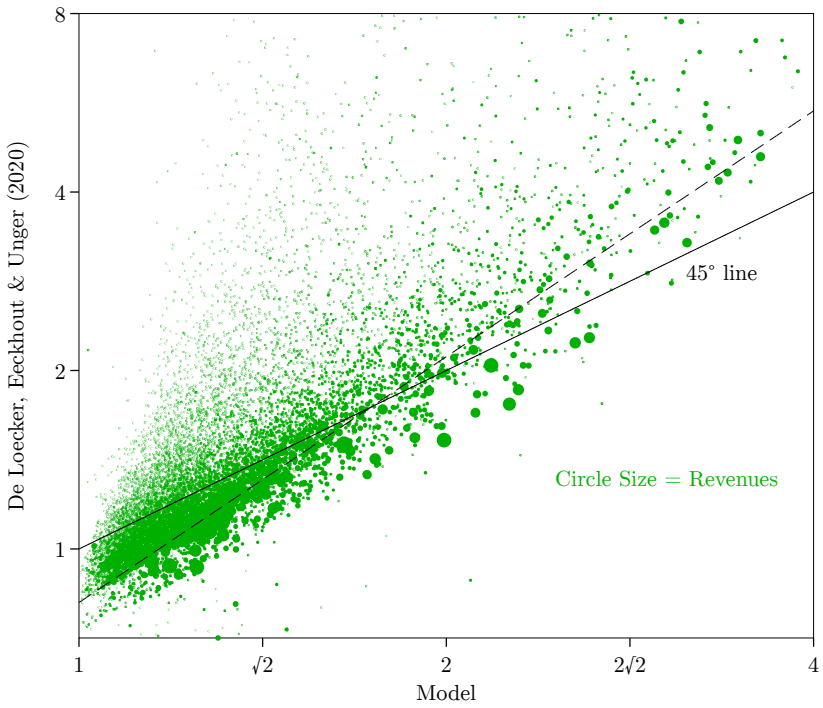

FIGURE NOTES: The figure plots the cross-section of markups obtained from the model against DEU's estimates with the size of the circle representing each firm's revenues.

DEU after 1982 (i.e., outside the calibration window).

Like Pellegrino (2019), we show that our aggregate markup series matches closely that of DEU, with one minor difference. In our model, we cannot compute markups without an estimate of $\mathbf{\Sigma}$ and $\mathbf{K}$. As a result, our markups series only starts in 1994.

Nonetheless, the markup implied by our model of oligopolistic competition under common ownership corresponds quite closely to the time trend of the average markup of DEU after 1994. In Figure 14 we plot the average markup based on our model (dark green line) and the average markup computed by DEU (light green line).

C.2 Cross-sectional Variation in Markups and Cross-Price Demand Elasticities

We now examine whether the cross-sectional variation in markups generated by our model matches that of DEU. Although matching DEU's cross-sectional markup variation tends to be a challenge for general equilibrium models with market power, our model-implied estimates perform quite well.

Figure 15 plots the cross-section of markups of our model against DEU's estimates using data from 1997, 2007 and 2017. As in DEU, we weight observations by revenues. Large circle sizes represent larger revenues and thus lead to larger weights. On the one hand, the graph shows that that there is more variation in the DEU markup estimates than in the model-based estimates: the

Table 6: Microeconometric Estimates of Cross-Price Demand Elasticities

Industry	Citation	Journal/Series
Auto	Berry, Levinsohn and Pakes (1995)	Econometrica
Beer	De Loecker and Scott (2016)	NBER Working Papers
Cereals	Nevo (2001)	Econometrica
Drugs	Chintagunta (2002)	Journal of Marketing Research
Laptops	Goeree (2008)	Econometrica

range is wider and the fitted regression slope (the dotted black line) is larger than one. On the other hand, our model-based estimates explain the cross-sectional variation in markups quite well. The R^2 of a regression of the DEU estimates on our model-based estimates with the slope constrained to one is equal to 88%.

We now investigate whether our model-based estimates of the cross-price elasticity of demand match the corresponding microeconometric estimates that were also used to calibrate the parameter α in Pellegrino (2019). Our strategy for calibrating α is to target microeconometric estimates from the IO literature. For a number of firm pairs we obtain direct estimates of the cross-price demand elasticity from empirical industrial organization studies described in Table 6. These estimates of the cross-price demand elasticity are then manually matched to the corresponding firm pair in Compustat. To calibrate α, we can rewrite equation (2.17) as:

$$\left|\frac{\partial \log p_i}{\partial \log q_j}\right| = \alpha \cdot \mathbf{a}_i' \mathbf{a}_j \frac{q_j}{p_i} \quad \forall\, i \neq j \tag{C.1}$$

Finally, for each firm pair, we can obtain an estimate of α by rearranging equation (C.1):

$$\hat{\alpha}_{ij} = \left|\frac{\partial \log p_i}{\partial \log q_j}\right| \Big/ \left(\mathbf{a}_i' \mathbf{a}_j \frac{q_i}{p_i}\right) \tag{C.2}$$

Equation (C.2) can be rearranged as

$$\log \left|\frac{\partial \log p_i}{\partial \log q_j}\right| \approx \log \alpha + \log \left(\mathbf{a}_i' \mathbf{a}_j \frac{q_j}{p_i}\right) \tag{C.3}$$

where we use the symbol $\approx$ to denote the fact that measurement error contaminates what should

Table 7: Model Fit — Cross-Price Demand Elasticities

Statistic	Data	Model	Targeted
Mean (in logs)	-2.81	-2.83	Yes
Standard Deviation (in logs)	0.522	0.326	No
Correlation with Data (in logs)	1.000	0.584	No
Number of Observations	32	32	

be a linear-in-logs relationship.

When calibrating α we can only choose the intercept of this linear relationship, but neither the slope nor the correlation. We use the (untargeted) slope and the goodness-of-fit of equation (C.3) to evaluate the model fit.

In Table 7, we report the relationship between the microeconometric estimates (i.e., the left side of equation C.3) from the industry studies listed in Table 6 and our own model-based estimates that use the calibrated value of α. Each observation is a firm pair (i, j) in one of the microeconometric industry studies.

Even though we cannot affect the slope of the relationship by calibrating α there is a strong positive correlation (0.584) between the industry study estimates and our own model-implied estimates. This is a remarkably high correlation especially because the microeconometric estimates come from industrial organization studies that use different econometric methodologies and employ entirely different assumptions about the underlying demand systems.

D Bertrand Oligopoly with Common Ownership

In this appendix we derive the Common Ownership Bertrand-Nash equilibrium under the simplifying assumption $\mathbf{\Delta} = \mathbf{0}$ (that is, under the assumption that all firms have a flat marginal cost function). We start by writing the vector of profit functions as a function of prices:

$$\pi \;=\; \text{diag}\left(\mathbf{p} - \mathbf{c}\right)\left[\mathbb{D}\left(\mathbf{b} - \mathbf{p}\right) + \mathbb{O}\left(\mathbf{b} - \bar{\mathbf{p}}\right)\right] \tag{D.1}$$

where $\mathbb{D}$ and $\mathbb{O}$ are, respectively, the matrices containing the diagonal and the off-diagonal elements of the matrix $\left(\mathbf{I} + \mathbf{\Sigma}\right)^{-1}$. We use the bar symbol ($\overline{\mathbf{p}}$) to indicate that firm i takes the prices of all other firms j as given. Then the system of first order condition is:

$$\mathbf{0} \;=\; \left[\mathbb{D}\left(\mathbf{b} - \mathbf{p}\right) + \mathbb{O}\left(\mathbf{b} - \bar{\mathbf{p}}\right)\right] - \mathbb{D}\mathbf{p} + \mathbb{D}\mathbf{c} \tag{D.2}$$

The Bertrand equilibrium is the fixed point $\mathbf{p} = \overline{\mathbf{p}}$. By imposing this equality and re-writing

this equation system in terms of the quantity vector ($\mathbf{q}$) we obtain:

$$\mathbb{D}\left(\mathbf{b}-\mathbf{c}\right) \;=\; \mathbf{q}+\mathbb{D}\left(\mathbf{I}+\boldsymbol{\Sigma}\right)\mathbf{q}+\left(\mathbf{K}\circ\mathbb{O}\right)\left[\mathbf{b}-\mathbf{c}-\left(\mathbf{I}+\boldsymbol{\Sigma}\right)\mathbf{q}\right] \tag{D.3}$$

We finally solve for the Bertrand equilibrium quantity vector $\mathbf{q}^{\mathrm{B}}$:

$$\mathbf{q}^{\mathrm{B}} \;=\; \left[\mathbf{I}+\mathbb{D}^{-1}+\boldsymbol{\Sigma}+\mathbb{D}^{-1}\left(\mathbf{K}\circ\mathbb{O}\right)\left(\mathbf{I}+\boldsymbol{\Sigma}\right)\right]^{-1}\left[\mathbf{I}+\mathbb{D}^{-1}\left(\mathbf{K}\circ\mathbb{O}\right)\right]\left(\mathbf{b}-\mathbf{c}\right) \tag{D.4}$$