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April 2022

https://equitablegrowth.org/working-papers/understanding-climate-damages-consumption-versus-investment/

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Understanding Climate Damages: Consumption versus Investment

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Abstract

Existing climate-economy models use aggregate damage functions to model the effects of climate change. This approach assumes climate change has equal impacts on the productivity of firms that produce consumption and investment goods or services. We show the split between damage to consumption and investment productivity matters for the dynamic consequences of climate change. Drawing on the structural transformation literature, we develop a framework that incorporates heterogeneous climate damages. When investment is more vulnerable to climate, we find short-run consumption losses will be smaller than leading models with aggregate damage functions suggest, but long-run consumption losses will be larger. We quantify these effects for the climate damage from heat stress and find that accounting for heterogeneous damages increases the welfare cost of climate change by approximately 4 to 24 percent, depending on the discount factor.

Keywords Climate Change, Structural Transformation, Growth

JEL Classification Codes 013, 044, Q56

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1 Introduction

Climate change affects the productivity of firms producing investment goods and services differently from the productivity of firms producing consumption goods and services. For example, construction firms produce investment goods and are more vulnerable to climate change than retail firms, which primarily produce consumption services. A long literature shows that distinguishing between consumption and investment productivity has important implications for understanding economic growth (e.g., Greenwood et al., 1997; Grossman et al., 2017), comparative development (e.g., Hsieh and Klenow, 2007, 2010), trends in inequality (e.g., Krusell et al., 2000; Grossman et al., 2021), and business cycles (e.g., Fisher, 2006; Justiniano et al., 2010). However, the existing macro literature on climate change abstracts from this distinction and instead uses aggregate damage functions that assume climate change has equal impacts on consumption and investment productivity (e.g., Nordhaus and Boyer, 2003; Golosov et al., 2014; Hassler et al., 2016, 2021; Barrage, 2020). We develop a dynamic general equilibrium framework that allows climate change to have different effects on consumption and investment productivity. Our framework reveals that the standard aggregate damage function approach can give misleading predictions about the impacts of climate change on macroeconomic dynamics and welfare.

Before describing our framework, it is helpful to briefly review the concept of an aggregate damage function, which is a key building block of existing macro climate-economy models. Climate change has heterogeneous impacts on different sectors (e.g., Auffhammer, 2018). The purpose of the aggregate damage function is to tractably combine these different impacts in order to study the dynamic effects of climate change on the macroeconomy. To build aggregate damage functions, researchers take the weighted average of more dissaggregated damages, with weights given by shares of gross domestic product (e.g., Nordhaus and Boyer, 2003; Barrage, 2020). The weighted average of damages is applied uniformly across the economy. A key issue with this approach is that the aggregation occurs outside of the model. As noted above, maintaining the distinction between damages to consumption and investment productivity inside the model is important for understanding the very dynamics that the macro climate-economy models are designed to study. Consumption in any period contributes directly to utility in that period, but has no impact on future outcomes.

In contrast, investment has no direct impact on utility in the current period. Instead, investment affects the future capital stock, which in turn, can be used to generate consumption and utility in later periods.

We first develop an analytic model to show the importance of heterogeneous damages. The analytic model has a Solow (1956)-like structure with a constant savings rate and separate consumption and investment goods. In this setting, the weights in the aggregate damage function are given by the consumption and investment shares of GDP. We show these weights yield misleading predictions for the effects of climate change on both short- and long-run consumption. In the short run, the effect of climate change on consumption is entirely determined by the climate damages to consumption productivity, instead of by the weighted average of damages. In the long run, because capital accumulates over time, the relative importance of damages to investment productivity depends on the capital share of GDP, and not on the smaller investment share, as assumed in the aggregate damage function. A direct consequence of applying incorrect weights is that the aggregate damage function will overstate short-run consumption losses and understate long-run consumption losses when investment is more vulnerable to climate change. The opposite implications hold if instead consumption is more vulnerable.

Building on the intuition from the analytic model, we next develop a quantitative model of structural change that accounts for heterogeneous damages. Our model extends the existing structural transformation literature in two ways (Herrendorf et al., 2013, 2014, 2021; Garcia-Santana et al., 2021). First, we model a more disaggregated economy that includes the construction and mining sectors, both of which are particularly vulnerable to climate change, as well as the standard agriculture, manufacturing, and services sectors. Second, we introduce climate damage into the model as sector-specific reductions in productivity. Output from the five sectors is used to produce final consumption and investment goods. Climate change endogenously has different effects on consumption and investment productivity because their sectoral compositions differ. For example, construction plays an important role in the production of investment and no role in the production of consumption. As a result, climate damage to construction productivity has a large impact on investment productivity and no impact on consumption productivity. We calibrate the parameters of the growth model to match the evolution of the sector shares of consumption and investment value added from 1947-2019 in the U.S. economy. The calibrated model fits the data closely.

We use our richer model to analyze the quantitative importance of heterogeneous damages for one prominent type of climate damage: labor productivity losses from heat stress. We focus on this particular application of our model because the damages from heat stress can be readily quantified from the existing literature. When humans undertake physically intensive tasks, the body must release heat to maintain a safe internal temperature. Rising temperatures from climate change make this physiological process more difficult, increasing the labor productivity losses in outdoor sectors, namely agriculture, mining, and construction. In the United States, outdoor work accounts for a greater share of investment value added, compared to consumption value added, largely because of construction. As a result, the climate damage from heat stress will have a greater impact on investment productivity. To parameterize the model, we take the relationship between heat stress and labor productivity from Dunne et al. (2013), who estimate this relationship from worker safety guidelines.[1]

To quantify the impacts of future climate change, we compare two simulations of our model: a "no-climate-change" scenario where climate remains constant after 2019 and a "climate-change" scenario where future temperature follows a path consistent with Representative Concentration Pathway (RCP) 8.5, which forecasts emissions in the absence of large-scale climate policy. To highlight the importance of heterogeneous damages, we compare the impact of climate change in our model with heterogeneous damages (HD model) to the impact in an otherwise equivalent model in which climate change equally affects productivity in consumption and investment. We refer to this second model as the DICE-like model because it follows the standard aggregate damage function approach pioneered by Nordhaus' DICE model (e.g., Nordhaus, 1993; Nordhaus and Boyer, 2003). We construct the aggregate damage function in the DICE-like model by taking a weighted average of the damages to the five sectors in

[1] The Occupational Safety and Health Administration (OSHA) and the U.S. military have consistent guidelines for the duration that various types of work can be safely performed at any given temperature. These guidelines, in turn, are based on outcomes from physiological studies (Dunne et al., 2013; Kjellstrom et al., 2018). The Biden Administration is currently planning to introduce new heat-related worker safety regulations: `https://www.nytimes.com/2021/09/20/climate/biden-heat-workplace-rules.html`.

the HD model, following the existing literature (Nordhaus and Boyer, 2003; Golosov et al., 2014; Barrage, 2020).

The HD model predicts climate change will lead to a larger fall in the long-run capital stock than the DICE-like model. For example, by 2200 climate change reduces the capital stock by 5.4 percent in the HD model, compared to only 1.2 percent in the DICE-like model. Intuitively, these differences arise because damages to investment productivity are larger than damages to consumption productivity. By applying the weighted average to all sectors, the aggregate damage function in the DICE-like model understates damage to investment productivity, resulting in the smaller predicted declines in capital. These results imply that the impact of climate change on the capital stock could be much larger than models with aggregate damage functions predict.

Similarly, accounting for heterogeneous damages also has important implications for predicting the short- and long-run effects of climate change on consumption. The HD model predicts that climate change will lead to smaller short-run decreases in consumption, but larger long-run decreases than the DICE-like model. The decrease in consumption caused by climate change depends on both the damage to consumption productivity and on the size of the capital stock. In the short run, the effect of climate change on the capital stock is relatively small, and thus the consumption losses are primarily determined by the damage to consumption productivity. The damage to consumption productivity is smaller in the HD model, leading to the smaller short-run losses. In the long run, the larger decrease in the capital stock in the HD model dominates, causing consumption losses in HD model to exceed those in the DICE-like model.

Our results have important implications for the welfare costs of climate change, as measured by the consumption equivalent variation in lifetime utility. Previous literature has established that the welfare costs of climate change are strongly sensitive to discount factors, because consumption losses occur far in the future (e.g., Dietz and Stern, 2008; Sterner and Persson, 2008). Compared to standard modeling approaches, accounting for heterogenous damage decreases short-run losses and increases long-run losses, making welfare costs even more sensitive to the discount factor. Using a discount factor derived from the behavior of market participants, the welfare costs are approximately 4 percent larger in the HD model than in the DICE-like model. Using instead a more Stern et al. (2006)-like discount factor that places greater weight on future outcomes implies that the welfare costs of the climate damage from heat stress are approximately 24 percent larger in the HD model. Ultimately, for any reasonable discount factor, the welfare cost of the climate damage from heat stress are larger than what leading models with aggregate damage functions would suggest (Nordhaus and Boyer, 2003; Golosov et al., 2014; Barrage, 2020).

We discuss two extensions of our core results. First, we simulate a more extreme scenario for future warming. Some scholars argue climate policy is best conceptualized as an insurance policy against the worst possible outcomes (e.g., Weitzman, 2009; Wagner and Weitzman, 2015). We find temperature realizations from the right tail of the distribution of possible climate outcomes increase the differences between the HD and DICE-like models. Second, we discuss how the HD model would compare to FUND and PAGE, the other two prominent models use by U.S. policymakers (Hope, 2006; Anthoff and Tol, 2014). These models implicitly assume climate change has no impact on investment productivity. Thus, comparing the HD model with DICE, which understates the impact of climate change on investment productivity but does not eliminate it, provides a lower bound on magnitudes of the differences between the HD model and FUND or PAGE.

Our paper is part of a long tradition of using growth models to study climate change (Nordhaus, 1993; Nordhaus and Boyer, 2003; Golosov et al., 2014; Barrage, 2020). Within this literature, there is a small but growing strand of work focusing on the relationship between climate and structural change. Desmet and Rossi-Hansberg (2015) study reallocation between agriculture and non-agriculture consumption sectors in a stylized model that includes both dynamics and spatial equilibrium. Casey et al. (2019) combine dynamic aspects of the stylized model with the endogenous fertility theory of Galor and Mountford (2008) to argue climate change will affect fertility rates and human capital accumulation. Nath (2020) quantifies consumption reallocation in a static model of international trade. While our model also includes structural change, our focus on damage heterogeneity across consumption and investment is new. Our model does include reallocation within consumption, but we find it is not quantitatively important in the United States. Consistent with this finding, the previous literature studying reallocation within consumption focuses on outcomes in developing countries.

The paper proceeds as follows. Section 2 provides motivating evidence on the different sectoral compositions of consumption and investment value added in the United States. Section 3 presents the analytic model, highlighting the importance of differentiating between climate damages to consumption and investment productivity. Section 4 describes the richer, quantitative model, and Section 5 explains the calibration. Section 6 presents the results of our quantitative analysis, and Section 7 concludes.

2 Empirical Motivation

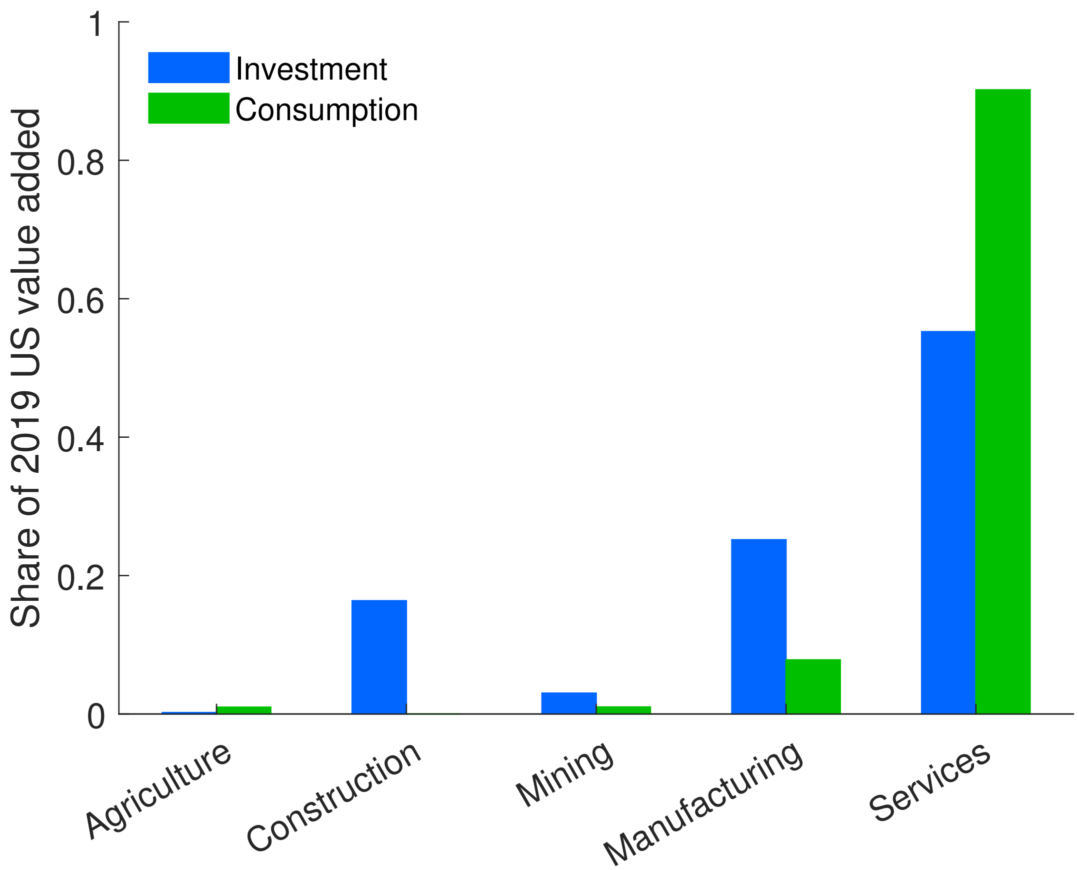
Climate damage has different effects on the productivity of consumption and investment because the sectoral composition of consumption and investment value added differ, and climate damages vary across sectors. Figure 1 divides the U.S. economy into five sectors: agriculture, construction, energy and mining, manufacturing, and services.[2] The blue and green bars plot the shares of value added in consumption and investment, respectively, produced in each of the five sectors. The data are derived from the National Income and Product Accounts for the U.S. economy in 2019 (see Appendix Section B.1). The figure highlights that the sectoral compositions of consumption and investment value added are different. For example, 90 percent of consumption value added is produced in the services sector, compared to only 60 percent of investment value added. If the damage from climate change varies across the five sectors, then the climate damage to consumption productivity will differ from the climate damage to investment productivity.

One prominent type of climate damage that varies across the different sectors is the loss in labor productivity due to heat stress. Of the five sectors, agriculture,

[2]Unlike most studies of structural change, we model a separate construction sector, which is particularly vulnerable to climate change. We have in mind the usual definition of construction: the creation or improvement of structures, which is a type of investment. In the data, some construction is classified as consumption. This type of consumption makes up less that one percent of all consumption value added. We re-classify this small subset of consumption as manufacturing, which is the standard approach to handling construction in the existing literature (Herrendorf et al., 2021; Garcia-Santana et al., 2021).

construction, and mining, are more vulnerable to climate damage from heat stress, because a substantial fraction of production in these sectors occurs outdoors and cannot easily be moved indoors. In contrast, production in manufacturing and services primarily occurs indoors. Firms in the indoor sectors can eliminate the climate damage from heat stress at low cost through the use of air conditioning (Nath, 2020). Figure 1 reveals that a much larger share of investment value added is produced in outdoor sectors, compared to consumption value added. All else constant, this comparison implies that the climate damage from heat stress will be larger for investment productivity than for consumption productivity.

Figure 1: Composition of Consumption and Investment Value Added



Note: The blue and green bars plot the fraction of U.S. investment and consumption value added, respectively, produced in each of the five sectors on the x-axis. The data are for 2019.

3 Simple Model and Intuition

Most existing climate-economy macro models are derived from the one-sector neoclassical growth model (e.g., Nordhaus and Boyer, 2003; Golosov et al., 2014; Barrage, 2020). The key dynamic equations are

$$
\begin{aligned}
Y_t^{\mathrm{Agg}} &= \left(D^{\mathrm{Agg}}(T_t) A_t N_t\right)^{1-\theta} K_t^{1-\theta}, && (1)\\
Y_t^{\mathrm{Agg}} &= C_t + X_t, && (2)\\
K_{t+1} &= X_t + (1-\delta) K_t, && (3)\\
U &= \sum_{t=0}^{\infty} \beta^t u(C_t), && (4)
\end{aligned}
$$

where $\theta,\ \beta \in (0,1)$, K_t and N_t are aggregate capital and labor, respectively, C_t is consumption, X_t is investment, Y_t^{Agg} is output, U is lifetime utility of the representative household, and $u(\cdot)$ is an increasing and concave period utility function. The variable T denotes the state of the climate, and the function $D^{\mathrm{Agg}}(T)$ captures the impact of climate on productivity, as shown in equation (1).[3] Equation (2) implies that one unit of the final good can always be transformed into one unit of either the consumption or investment good, regardless of climate damages. Thus, this framework assumes that climate change has an equal impact on the productivity of producing the consumption and investment goods. The evidence presented in Section 2, however, suggests that climate damages to investment and consumption productivity differ. Equations (3) and (4) highlight the different roles that consumption and investment play in economic dynamics. Consumption in any period contributes to utility directly, but has no impact on future economic outcomes. Investment has no direct impact on utility in the current period, but it influences future production levels, which can in turn be used to generate consumption and utility in later periods.

To analytically explore the different consequences of climate damages to consumption and investment productivity, we build a simple Solow (1956)-like growth model with separate consumption and investment goods. We show that, for a given aggregate damage function, the consequences of climate change depend on how damages

[3]These models also specify the links between climate change and economic activity. We take climate change as exogenous and focus on the consequences, rather than the causes, of climate change.

are split between consumption and investment.

Consumption (C) and investment (X) are produced from Cobb-Douglas production functions,

$$C_t = \left(D_C(T) A_C n_{Ct}\right)^{1-\theta} k_{Ct}^{\theta} \quad \text{and} \quad X_t = \left(D_X(T) A_X n_{Xt}\right)^{1-\theta} k_{Xt}^{\theta}, \tag{5}$$

where for each $J \in \{C, X\}$, A_J is technology, n_{Jt} is labor inputs, and k_{Jt} is capital. Again, T captures the state of the climate, and $D_J(T)$ is now a consumption- or investment-specific damage function. With perfect competition, θ is equal to the capital share of income. Following Herrendorf et al. (2014, 2021), θ is common to both production functions. As in equation (3), capital accumulates linearly from investment, with depreciation rate δ. There is free mobility of factors of production and perfect competition. For expositional simplicity, we abstract from technological progress and population growth. The capital and labor market clearing conditions are $K_t = k_{Ct} + k_{Xt}$ and $n_{Xt} + n_{CT} = 1$. We assume that the savings rate, $s \in (0, 1)$, is constant. Given that the economy is closed, the savings rate is also the investment share of final expenditure.[4] Investment is the numeraire, and P_t is the relative price of consumption. For the analytic results, we model climate change as a one-time permanent increase in temperature, T. We focus on the empirically relevant case where $\theta > s$ and $D_J'(\cdot) < 0$ for $J \in \{C, X\}$.

Consistent with the approach used in most climate-economy models, we define the aggregate damage function as the weighted average of damage to consumption and investment productivity with the weights equal to the shares of output (e.g., Nordhaus and Boyer, 2003; Barrage, 2020):[5]

$$D^{\text{Agg}}(T) \equiv D_X(T)^s D_C(T)^{1-s}. \tag{6}$$

To understand the effects of climate change, we take the derivative of the log of the

[4] We treat the two production functions as capturing final expenditure on consumption and investment. The model is unchanged if we consider value-added production functions with s equal to the constant share of value-added devoted to investment. In our quantitative application, we focus on value-added production functions, which makes it easier to parameterize climate damages.

[5] We focus on the geometric mean for analytic convenience.

aggregate damage function with respect to temperature:

$$\frac{d\ln D^{\text{Agg}}(T)}{dT} = s\frac{d\ln D_X(T)}{dT} + (1-s)\frac{d\ln D_C(T)}{dT}. \tag{7}$$

Thus, aggregate damage functions imply that the impacts of climate change on productivity are fully captured by the weighted average of the changes in damages to consumption and investment productivity.

To help with the underlying intuition, panel (a) of Figure 2 plots a level set of the effect of climate change on productivity, defined by the derivative of the log of the aggregate damage function in equation (7). At every point on the line, the effect of climate change on the aggregate damage function is constant, $\left|\frac{d\ln D^{\text{Agg}}(T)}{dT}\right| \equiv \bar{D}$, but the effects on consumption and investment productivity differ. Standard climate economy models assume that the consequences of climate change should be identical for any point on the line in panel (a) of Figure 2, regardless of the split between consumption and investment damages.

We first show that the impact of climate change on short-run consumption depends on how climate change differentially effects consumption and investment productivity. In other words, the impact of climate change on short-run consumption depends on where the economy lies along the line in panel (a) of Figure 2. Following the standard steps for structural change growth models (e.g., Herrendorf et al., 2014), it is straightforward to show that the relative price of consumption is the inverse of the relative productivity between the two sectors,

$$P_t = \left(\frac{A_X D_X(T)}{A_C D_C(T)}\right)^{1-\theta}.$$

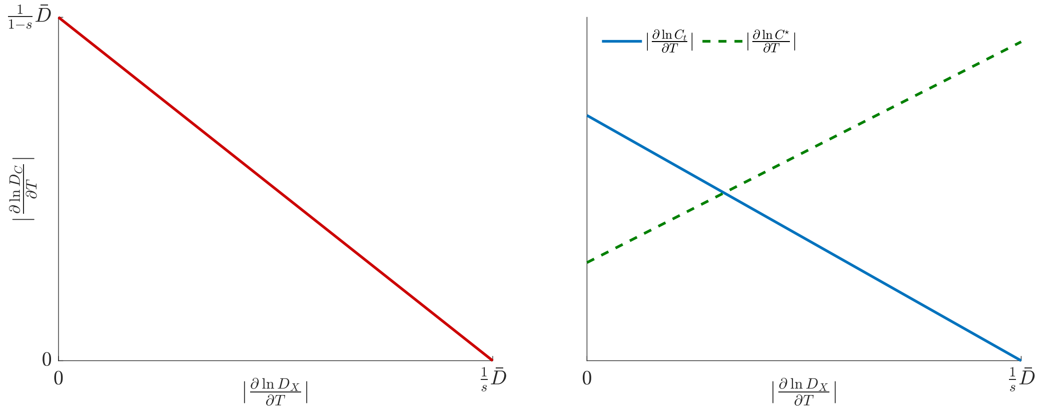
Also, gross domestic product is the sum of investment and consumption expenditure:

$$Y_t \equiv X_t + P_t C_t = \left(A_X D_X(T)\right)^{1-\theta} K_t^{\theta},$$

where the rightmost equality follows from combining the production functions for consumption and investment, the market clearing conditions, and the expression for the relative price of consumption.

The exogenous savings rate implies that $P_t C_t = (1-s)Y_t$. Using this relation-

Figure 2: Heterogeneous Climate Impacts in the Analytic Model



(a) Level set of aggregate damages

(b) Consumption along the level set

Note: Panel (a) shows a level set of the derivative of the aggregate damage function, $\left|\frac{d \ln D^{\mathrm{Agg}}(T)}{dT}\right| = \bar{D}$. This impact is the same at every point on the solid red line, but the split between damage to consumption and investment productivity differs. Panel (b) plots the consumption impacts of climate change at different points along the level set in panel (a). The solid blue line in panel (b) shows the magnitude of the effect of climate change on short-run consumption along the level set, as captured by equation (8). The dashed green line shows the magnitude of the effects of climate change on long-run consumption along the level set, as captured by equation (9). The figure demonstrates that when damages are more concentrated in investment, the impact of climate change on long-run consumption is larger and on short-run consumption is smaller.

ship, as well as the expressions for GDP and the relative price of capital, yields the equilibrium value of consumption in period t,

$$C_t = (1-s)\left(A_C D_C(T)\right)^{1-\theta} K_t^{\theta}.$$

Taking the derivative of the log of C_t with respect to temperature, T, yields the impact of climate change on short-run consumption:

$$\frac{\partial \ln C_t}{\partial T} = (1-\theta)\frac{\partial \ln D_C(T)}{\partial T}. \tag{8}$$

Only damages to consumption productivity affect the level of consumption in the short run (i.e., conditional on a value of aggregate capital, K_t). Holding constant the effects of climate change on the aggregate damage function, $\left|\frac{d \ln D^{\mathrm{Agg}}(T)}{dT}\right|$, climate change has a larger impact on short-run consumption when the effect of climate change on consumption productivity, $\left|\frac{\partial \ln D_C(T)}{\partial T}\right|$, is large (which in turn implies that and the effect of climate change on investment productivity, $\left|\frac{d \ln D_X(T)}{dT}\right|$, is small). In the context of panel (a) of Figure 2, as the economy moves from left to right along the level set, the effect of climate change on short-run consumption falls. The solid blue line in panel (b) in Figure 2 graphically summarizes this result. It plots the magnitude of the effect of climate change on short-run consumption, $|\frac{\partial \ln C_t}{\partial T}|$, holding $\left|\frac{d \ln D^{\text{Agg}}(T)}{dT}\right|$ fixed at $\bar{D}$. As the economy moves from left to right along the horizontal axis, the effect of climate change on investment productivity rises and the effect of climate change on consumption productivity falls. As a result, the effect of climate change on short-run consumption decreases.

We next show that the impact of climate change on steady state consumption also depends on how aggregate damages are split between consumption and investment productivity. We use asterisks ($*$) to denote steady state values. Following the standard steps for the Solow model, the steady state level of capital is

$$K^* = (A_X D_X(T)) \left(\frac{s}{\delta}\right)^{\frac{1}{1-\theta}}.$$

Steady state capital only depends on damages to investment productivity, highlighting once again the different roles that damages to consumption and investment productivity play in the economy's dynamic response to climate change.

To solve for steady state consumption, we combine the steady state level of capital with the assumption of a constant savings rate and the expression for P_t,

$$C^* = (1-s)\left(\frac{s}{\delta}\right)^{\frac{\theta}{1-\theta}} (A_X D_X(T))^{\theta} (A_C D_C(T))^{1-\theta}.$$

Taking the derivative of the log of C^* with respect to temperature, T, yields the impact of climate change on long-run consumption:

$$\frac{\partial \ln C^\star}{\partial T} = \theta \frac{\partial \ln D_X(T)}{\partial T} + (1-\theta)\frac{\partial \ln D_C(T)}{\partial T}. \tag{9}$$

The effect of climate change on long-run consumption depends on its effects on both consumption and investment damage. Reductions in investment productivity decrease the size of the economy, and reductions in consumption productivity decrease consumption conditional on the size of the economy. The dashed green line in panel (b) of Figure 2 plots the magnitude of the effect of climate change on long-run consumption, $|\frac{d\ln C^*}{dT}|$, for different points on the level set in panel (a). As the economy moves from left to right along the horizontal axis in panel (b), the effect of climate change on investment productivity rises and the effect on consumption productivity falls. The figure demonstrates that the impact of climate change on steady state consumption is larger when damages are more concentrated in investment.

To highlight the relevant intuition, consider the effects of climate change on long-run consumption at the horizontal intercept from panel (a) of Figure 2, where climate change only affects investment productivity and at the vertical intercept, where climate change only affects consumption productivity. If climate change only affects consumption productivity then $\left|\frac{d\ln D_C(T)}{dT}\right| = \bar{D}/(1-s)$ and $\left|\frac{d\ln C^*}{dT}\right| = \left(\frac{1-\theta}{1-s}\right)\bar{D}$. Instead, if climate change only affects investment productivity, then $\left|\frac{d\ln D_X(T)}{dT}\right| = \bar{D}/s$ and $\left|\frac{d\ln C^*}{dT}\right| = \left(\frac{\theta}{s}\right)\bar{D}$. Thus, as long as $\theta \neq s$, the effect of climate change on long-run consumption, $\left|\frac{d\ln C^*}{dT}\right|$, depends on how climate damages are split between consumption and investment. In practice, $\theta > s$, implying that, for a given change in the aggregate damage function, climate change has bigger impacts on steady state consumption when damages are concentrated in investment.

Standard climate economy models implicitly apply an aggregate damage function to all sectors, where the aggregate damage function is a weighted average of more disaggregated damages. In the context of our simple model, the aggregate damage function is the weighted average of the damage to consumption and investment productivity, with weights equal to the consumption and investment shares of contemporaneous output, as in equation (6). While convenient, these weights do not correctly capture how damages affect consumption in either the short or the long run. In the short run, only consumption damages affect the level of consumption. Thus, applying the weighted average to all sectors overstates the importance of investment damages for contemporaneous consumption. In the long run, the weight of investment damages for steady state consumption, θ, is greater than their weight in contemporaneous output, s, because damages to investment productivity compound over time.[6] Consequently, the aggregate damage function weight of s on investment

[6] To see the role of compounding, note that the elasticity of consumption to $D_C(T)$ is $1-\theta$ for

damages understates the impact of investment productivity losses on steady state consumption.

4 Quantitative Model

Building on the intuition from Section 3, we next develop a richer model that accounts for heterogeneous damages and is amenable to quantitative analysis. In particular, we consider a finite time horizon, endogenize the savings rate, and model consumption and investment as goods that are produced from the value added of the five underlying sectors in Figure 1. Climate change directly affects productivity in these five sectors. The richer model draws on the generalized structural transformation framework of Herrendorf et al. (2013, 2014, 2021) and Garcia-Santana et al. (2021). We build on their work by (i) adding climate change as a determinant of productivity and (ii) modeling a more disaggregated economy that includes the construction and mining sectors, both of which are particularly vulnerable to climate change.

Production. Production is perfectly competitive. There are five production sectors with positive value added: agriculture (a), services (s), construction (b), energy and mining (e), and manufacturing (m). We use j to index these sectors.

Each sector has a representative firm with a Cobb-Douglas production function:

$$y_{jt} = k_{jt}^{\theta} \left(D_j(T_t) A_{jt} n_{jt}\right)^{1-\theta}, \tag{10}$$

where y_{jt} is the output from sector j at time t, n_{jt} is the quantity of labor inputs, A_{jt} is the productivity of technology, and k_{jt} is the quantity of capital used in production. Damage function $D_j(T_t)$ is specific to sector j. The argument, T_t, is a measure of climate, which evolves exogenously. Damage functions take values in the interval $[0, 1]$, with 1 representing no damage from climate. Consistent with existing models

both contemporaneous and steady-state consumption. The impact of consumption productivity on consumption is the same in both the short and long run, because consumption productivity has no impact on the dynamics of the economy. Meanwhile, the elasticity of consumption with respect to $D_X(T)$ depends entirely on capital accumulation. This elasticity grows from 0 in the short run, when there is no change in capital, to θ in the steady state, which is precisely the elasticity of output with respect to capital.

(e.g., Herrendorf et al., 2014), we assume that θ is the same across all sectors. Herrendorf et al. (2015) show that differences in θ across sectors do not have first order consequences for structural change (see also Gollin et al., 2014). The productivity of technology grows at a constant, exogenous, and sector-specific rate,

$$A_{jt} = (1 + \gamma_j)A_{jt-1}. \tag{11}$$

Output from sector j, y_{jt}, can be used to produce consumption or investment. Market clearing for output from sector j is given by

$$y_{jt} = c_{jt} + x_{jt}, \tag{12}$$

where c_{jt} is the quantity of sector j output that is used to produce consumption and x_{jt} is the quantity of sector j output that is used to produce investment.

A representative firm produces a final investment good (X_t) by combining value added from the construction sector with value added from the other sectors according to a nested CES production function. The production function is:

$$X_t = \left[\xi_b^{\frac{1}{\sigma_x}} x_{bt}^{\frac{\sigma_x - 1}{\sigma_x}} + \xi_z^{\frac{1}{\sigma_x}} x_{zt}^{\frac{\sigma_x - 1}{\sigma_x}} \right]^{\frac{\sigma_x}{\sigma_x - 1}}, \tag{13}$$

where $\xi_b + \xi_z = 1$ and

$$x_{zt} = \left(\sum_{j \neq b} \xi_j^{\frac{1}{\sigma_z}} x_{jt}^{\frac{\sigma_z - 1}{\sigma_z}} \right)^{\frac{\sigma_z}{\sigma_z - 1}}, \ \sum_{j \neq b} \xi_j = 1, \tag{14}$$

is the inner CES function.

Factor Market Clearing. There is no population growth and labor is supplied inelastically. We normalize the size of the labor force to one:

$$1 = \sum_j n_{jt}, \tag{15}$$

with $n_{jt} \geq 0 \ \forall j, t$. Aggregate capital, K_t, can be costlessly moved between sectors.

Market clearing for capital goods is given by

$$K_t = \sum_j k_{jt}, \tag{16}$$

with $k_{jt} \geq 0 \ \forall j, t$. Capital accumulates according to

$$K_{t+1} = X_t + (1-\delta)K_t, \tag{17}$$

where $\delta \in (0,1)$ is the depreciation rate.

Individuals. There is a representative household with generalized Stone-Geary preferences (Herrendorf et al., 2013). Flow utility is defined over the four consumption categories – c_{at}, c_{st}, c_{et} and c_{mt} – according to:

$$C_t = \left(\sum_{j \neq b} \omega_j^{\frac{1}{\sigma_c}} (c_{jt} + \bar{c}_j)^{\frac{\sigma_c - 1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c - 1}}, \ \sum_{j \neq b} \omega_j = 1. \tag{18}$$

For each j, $\omega_j > 0$ is a time-invariant weight in the utility function, and $\bar{c}_j$ governs the size of income effects. The term $\sigma_c > 0$ is closely related (but not exactly equal) to the elasticity of substitution between sectors. We refer to C_t as aggregate consumption. Garcia-Santana et al. (2021) show that generalized Stone-Geary preferences can recreate the stylized facts of structural change when combined with structural change in investment, as in our framework.[7]

Lifetime utility of the representative household is given by

$$U = \sum_{t=0}^{t_{\max}} \beta^t \frac{C_t^{1-\chi}}{1-\chi}, \tag{19}$$

where $\beta \in (0,1)$ is the time discount factor, χ is the elasticity of intertemporal substitution, and $t_{\max}$ is the final period.[8] This implies that the representative household

[7] In models that do not separate structural change in investment, the Stone-Geary function has insufficiently strong income effects to match patterns in the data. Recent work has developed alternative utility functions with stronger income effects to explain economy-wide level of structural change when there is no asymmetry between consumption and investment (Boppart, 2014; Alder et al., forthcoming; Comin et al., 2021),

[8] In the quantitative application, we choose $t_{\max}$ to be sufficiently far in the future such that

discounts future consumption losses from climate damage because of β, which captures the pure rate of time preference, and also χ, which governs changes in the marginal utility of aggregate consumption. We normalize the price of the final investment good to one. The budget constraint of the representative household is given by

$$\sum_{j \neq b} p_{jt} c_{jt} + X_t \leq w_t L_t + (1 + r_t - \delta) K_t, \tag{20}$$

where p_{jt} is the price of value added from sector j, w_t is the wage rate, and r_t is the rental rate.

Analysis. As noted above, our model adds climate damages and a higher level of disaggregation to the existing literature on structural transformation (e.g., Herrendorf et al., 2013, 2014, 2021; Garcia-Santana et al., 2021). Conveniently, the solution techniques from the earlier literature still apply with these new elements. Details of the analysis are in Appendix A.

5 Calibration

A key feature of our analysis is allowing climate damage to vary between consumption and investment productivity. To highlight the importance of this heterogeneity, we calibrate the model for one particular type of climate damage: labor productivity losses due to heat stress. We choose heat stress because we can use existing estimates from the climate science literature and there is a natural distinction between the vulnerability of indoor and outdoor sectors.

The time period in the model is one year. We estimate the parameters of the CES investment and utility functions from historical data on sector shares and relative prices. We take the remaining parameters of the growth model directly from the data and the existing literature.

changing this endpoint has no impact on the outcomes we study.

5.1 Labor Productivity Losses From Heat Stress

Following our discussion from Section 2, we assume that the outdoor sectors of agriculture, energy and mining, and construction are equally susceptible to the climate damage from heat stress, while the indoor sectors of manufacturing and services are not vulnerable. Consistent with this assumption, Nath (2020) estimates the impact of temperature on productivity in the U.S. manufacturing sector and finds zero effect. While he also finds that manufacturing firms increase energy expenditures on very hot days, the cost of doing so is sufficiently small that it has negligible impacts on productivity.

Our goal is to predict the labor productivity loss in outdoor sectors from heat stress. This loss depends on workplace rules and norms, as well as individual worker circumstances. Worker safety organizations, the U.S. military, and the American College of Sports Medicine all provide guidelines for how much effort individuals can safely exert under different climate conditions (Armstrong et al., 2007; Dunne et al., 2013).[9] Dunne et al. (2013) use these guidelines to estimate the labor-productivity losses from extreme heat. We use their estimates to calculate the climate damage from heat stress in the outdoor sectors in our model.

Specifically, Dunne et al. (2013) estimate the fraction of a standard eight-hour work day for which an individual can safely sustain the effort needed to engage in "heavy work," which is typical of agriculture and construction (e.g., ILO, 2019, p. 91). For example, if under given temperature conditions, an individual could only safely work for six hours, then the damage function would apply a 25 percent reduction in labor productivity. The safety limits are taken from the U.S. military and from the occupational safety guidelines, both of which build on physiological studies (ACGIH, 1996; Parsons, 2006; Army, 2003).[10]

Dunne et al. (2013) show that the occupational safety organizations and the U.S.

[9] The labor productivity loss is different than changes in the maximum effort that workers could exert at a given temperature without assuring negative consequences. The guidelines are designed to protect workers of varying levels of health from meaningful risk of heat stress.

[10] For an example of such guidelines, see the Heat Stress Work/Rest Schedule for mining industries from the National Institute for Occupational Safety and Health (NIOSH), which is available at https://www.cdc.gov/niosh/mining/UserFiles/works/pdfs/2017-127.pdf. According to the guidelines, "heavy work" includes "climbing", "Carrying equipment/supplies weighing 40 pounds or more", and "using hand tools (shovel, fin-hoe, scaling bar) for extended periods."

military have a consistent set of heat stress guidelines, which suggest that the relationship between labor productivity and heat stress is given by:

$$\text{Fraction of Labor Productivity Lost} = 0.25 \cdot \max[0, (WBGT_t - 25)^{\frac{2}{3}}], \tag{21}$$

with an upper bound of one. The $WBGT_t$ is the wet bulb globe temperature, which incorporates the ambient air temperature, humidity, wind speed, and solar irradiance to capture the climatic conditions the body actually experiences. This is the standard measure of heat exposure used in the extensive physiology and occupational safety literature on heat stress (e.g., Hsiang, 2010; Kjellstrom et al., 2018).

The labor-productivity losses specified in equation (21) are zero for all WBGT below 25°C. As the WBGT increases beyond 25°C, the fraction of labor productivity lost increases, until it reaches its upper bound of one at 33°C. If the WBGT exceeds 33°C, the guidelines suggest that it is not safe to perform any outdoor work. These results are quite similar to the guidelines from the American College of Sports Medicine, which suggest that some athletes should reduce physical exertion when the WBGT exceeds 22.3°C and that all exercises should be cancelled if the WBGT exceeds 32.3°C, even for “acclimatized, fit, low-risk individuals” (Armstrong et al., 2007, Table 2).

Of course, the worker-safety guidelines for heat stress do not perfectly reflect the labor-productivity losses from extreme heat. Some workers could reduce effort by more than what the guidelines recommend. Others could work beyond what the guidelines recommend, which could either increase or decrease their overall productivity, depending on the consequences of heat stress. But the consistency of the guidelines across a wide range of organizations suggests that they provide an empirically founded, reasonable measure of how changes in wet bulb globe temperature will affect labor productivity in the United States.

5.2 Climate Damage From Heat Stress

To project the future consequences of heat stress for the U.S. economy, we proceed with three steps. First, we project the daily distribution of the WBGT for each U.S. county for the period 2020–2100. Second, we combine these daily projections with the specification for lost labor productivity in equation (21) to project annual county level damages. Third, we average annual damages across counties to project national climate damages from heat stress.

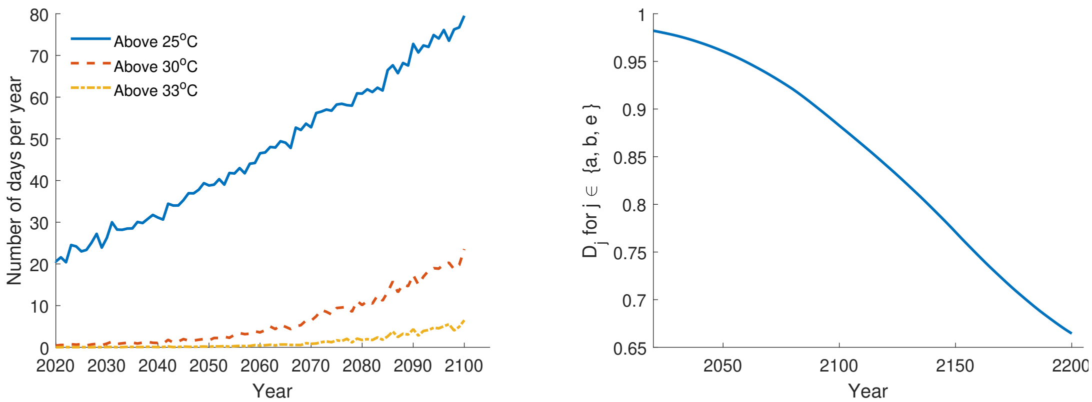
We first describe how we construct the county-level projections of the WBGT distribution. Rasmussen et al. (2016) report county-level projections of the annual distribution of average daily temperature through year 2100 under different representative concentration pathways (RCPs) for each climate model in the Coupled Model Intercomparison Project (CMIP) archive.[11] Additionally, the data include projections from model surrogates that populate the right tail of the distribution of global mean surface temperature from 2080-2100. To aggregate across individual climate models, we follow Rasmussen et al. (2016) and Hsiang et al. (2017) and take a weighted average of the individual model projections, where the weights are determined by the relative probabilities that a given climate model represents the true outcome. For our analysis we use the RCP 8.5 projections, which are designed to approximate global emissions in the absence of large-scale climate policy. To calculate the county-level distribution of the WBGT, we combine the projected RCP 8.5 county-level temperature distributions with information on relative humidity, solar irradiance and wind speed (see Appendix Section B.3).

To provide a sense of how climate change will affect the number of days that workers are vulnerable to heat stress, the panel (a) of Figure 3 plots the population-weighted average of the number of days each year that the WBGT is above 25°C, above 30°C and above 33°C. As shown in equation (21), 25°C is threshold beyond which workers begin to loose labor productivity from heat stress. Similarly, 33°C is the threshold beyond which all labor productivity is lost to heat stress. Climate change leads to large increases in the number of days for which the WBGT is above these threshold values, increasing the damage from heat stress. For example, the number of days above 25°C quadruples between 2020 and 2100, from 20 to 80 days.

We combine the county-level projections for the daily distribution of WBGT and the function for the fraction of labor productivity lost in equation (21) to determine the daily proportional loss in labor productivity from heat stress in outdoor sectors for each county-year. To calculate annual county-level damages, we average across

[11] `https://www.wcrp-climate.org/wgcm-cmip`.

Figure 3: Climate Change Projections



(a) Wet Bulb Globe Temperature (b) Climate Damage From Heat Stress

Note: panel (a) plots the population-weighted number of days per year that the WBGT exceeds 25°C (solid blue line), 30°C (dashed orange line), and 33°C (dotted yellow line). Panel (b) plots the climate-change component of labor productivity in outdoor sectors. Subscripts a, b and e denote the agriculture, construction and mining sectors, respectively.

daily productivity losses in each county-year.

To aggregate to the national level, we take the average of the annual county-level damages from heat stress, weighted by 2019 total county population. This process generates national annual damages to outdoor sectors through 2100, which is when the county-level temperature projections end. This damage series has two shortcomings. First, it has short-run fluctuations from variation in weather that are unrelated to trends in climate. Second, it cannot account for the effects of climate change beyond 2100. To address both of these issues, we use a second degree polynomial to capture the relationship between annual U.S. damages to outdoor sectors and global CO_2 concentrations under RCP 8.5 from 2020-2100. The fitted relationship smooths out the short-run fluctuations. We combine the fitted relationship with the longer available time series for projections of global CO_2 concentrations to extend our damages estimates through year 3000. Under RCP 8.5, CO_2 concentrations are constant after 2250.

Panel (b) of Figure 3 plots the function for the projected climate damage from heat stress for outdoor sectors over the 2020-2200 period, equal to D_j in the model for all outdoor sectors, namely agriculture (a), construction (b), and energy (e). A value of $D_j = 1$ would imply no climate damage from heat stress. This is the value applied to the indoor sectors of manufacturing (m) and services (s). As D_j falls below one, the climate damage from heat stress increases. The vertical intercept implies that heat stress reduced labor productivity by 2 percent in 2020. Between 2020 and 2200, D_j falls from 0.98 to 0.66, implying that future changes in climate increase the losses in outdoor-sector labor productivity from 2 percent to 44 percent.

5.3 Growth model

We estimate the utility function parameters from data on relative prices and consumption value added, closely following the procedure outlined in Horowitz et al. (2006), Herrendorf et al. (2013), and Garcia-Santana et al. (2021). All data on sector-level prices and quantities come from the Bureau of Economics Analysis (BEA). We first derive expressions for the sector shares of consumption value added,

$$\frac{p_{jt}c_{jt}}{\sum_{j\neq b} p_{j,t}c_{j,t}} = \frac{\omega_j p_{j,t}^{1-\sigma_c}}{\sum_{j\neq b}\omega_j p_{jt}^{1-\sigma_c}} \cdot \left(1 + \frac{\sum_{j\neq b} p_{j,t}\bar{c}_{j,t}}{\sum_{j\neq b} p_{j,t}c_{j,t}}\right) - \frac{p_{j,t}\bar{c}_j}{\sum_{j\neq b} p_{j,t}c_{j,t}}, \tag{22}$$

$j = a, e, m, s$, from the model first order conditions (see Appendix Section A.1.5). We use iterated feasible generalized nonlinear least squares to estimate the resulting demand system from U.S. data on relative prices and consumption value added in each sector (constructed from the BEA input-output accounts, see Appendix Section B.1). The shares sum to unity, causing the error covariance matrix to be singular. Therefore, we drop the sector share of agriculture when we do the estimation. Additionally, several of our parameter values are constrained. In particular, the substitution elasticities must be non-negative and the CES weights must sum to unity. We transform the constrained parameters into unconstrained parameters,

$$\begin{aligned}\sigma_c = e^{q_0},\ \omega_a &= \frac{1}{1+\sum_{i=1}^3 e^{q_i}},\ \omega_g = \frac{e^{q_1}}{1+\sum_{i=1}^3 e^{q_i}}, \\ \omega_m &= \frac{e^{q_2}}{1+\sum_{i=1}^3 e^{q_i}},\ \omega_s = \frac{e^{q_3}}{1+\sum_{i=1}^3 e^{q_i}}.\end{aligned} \tag{23}$$

We estimate the demand system in terms of the unconstrained parameters, $q_0, q_1, q_2, q_3, \in (-\infty, \infty)$ and $\bar{c_a},\ \bar{c_g},\ \bar{c_s} \in (-\infty, \infty)$. Using the unconstrained parameter estimates,

Table 1: Consumption and Investment Parameter Estimates

Panel A. Consumption

σ_c	$\bar{c}_a$	$\bar{c}_g$	$\bar{c}_s$	ω_a	ω_g	ω_m	ω_s
0.21	-0.21	-0.45	6.91	0.005	0.004	0.08	0.91
(0.007)	(0.004)	(0.012)	(0.146)	(0.0002)	(0.0001)	(0.001)	(0.001)

Panel B. Investment

σ_x	σ_z	ζ_b	ζ_z	ξ_a	ξ_g	ξ_m	ξ_s
1.01	0.21	0.16	0.84	0.01	0.06	0.32	0.61
$(\cdot)$	(0.020)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.003)

Note: Panels A and B report the estimated parameter values for consumption and investment, respectively. Robust standard errors are in parentheses.

we calculate the point estimates and standard errors on the constrained parameters, $\sigma_c,\ \omega_a,\ ,\omega_g,\ \omega_m,\ \omega_s$, using the delta method.

Our procedure to estimate the CES investment parameters parallels our procedure for consumption. We derive expressions for the sector shares of investment value added,

$$\frac{p_{b,t}x_{b,t}}{X_t} = \xi_x p_{bt}^{1-\sigma_x}, \tag{24}$$

$$\frac{p_{j,t}x_{j,t}}{X_t} = (1-\xi_x)\xi_j p_{jt}^{1-\sigma_j}\left(\sum_{j\neq b}\xi_z p_{x_jt}^{1-\sigma_z}\right)^{\frac{\sigma_z-\sigma_x}{1-\sigma_z}},\ j\neq b, \tag{25}$$

and use iterated feasible nonlinear least squares to estimate the resulting demand system. We again drop the sector-share of agriculture from the estimation and transform the constrained substitution elasticities and weights into unconstrained parameters, as in equation (23).

Table 1 reports the estimated parameter values with heteroskedasticity-robust standard errors in parentheses. Consistent with earlier studies (e.g., Herrendorf et al., 2013; Garcia-Santana et al., 2021), we find that the calibrated substitution elasticities for consumption and the inner-most investment nest are close to zero. The substitution elasticity for the outer investment nest is close to one. Additionally, the signs of the non-homothetic terms mirror the patterns from earlier work, $\bar{c}_a < 0$ and $\bar{c}_s > 0$ (Herrendorf et al., 2013).

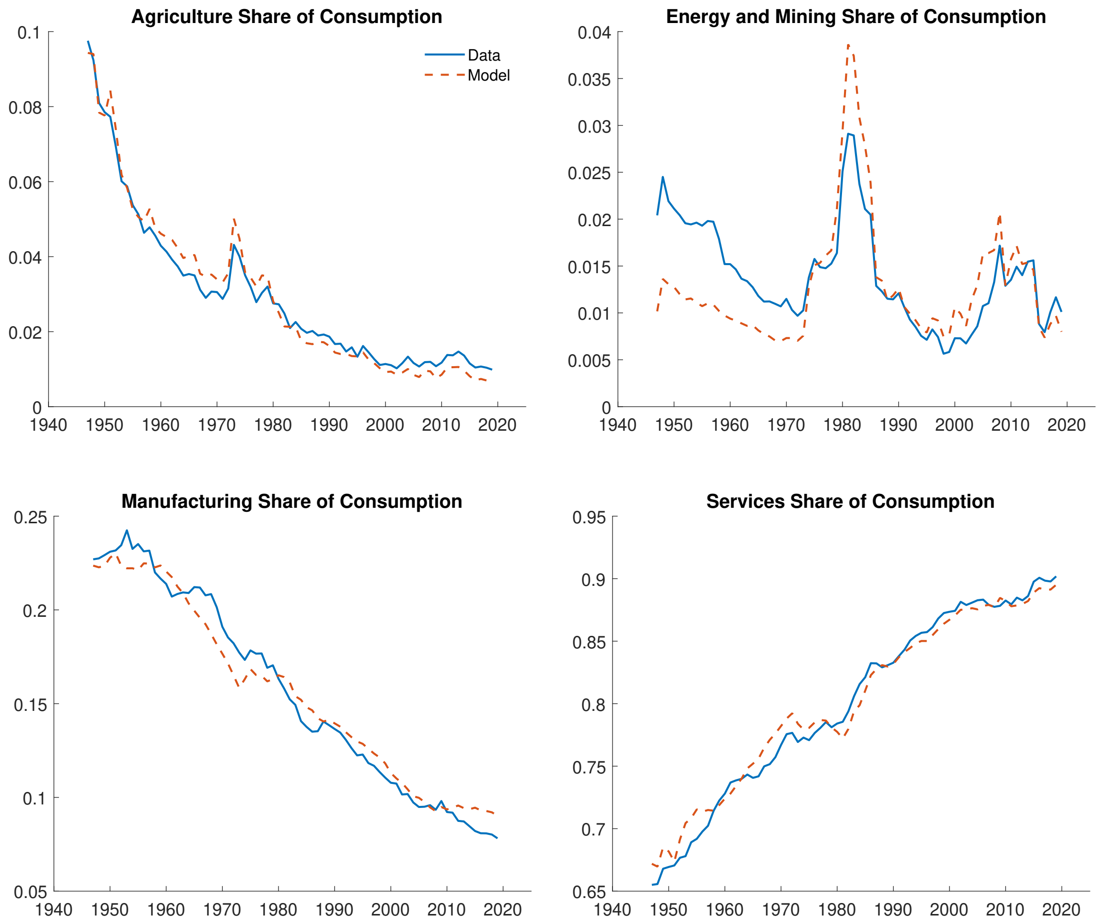
Figures 4 and 5 plot the model and empirical sector shares of consumption and investment value added, respectively.[12] Overall, the model fits these sector shares quite well. However, the model only matches the trend in the construction share. It does not match the volatility. Since our model is designed to capture long-run trends, it necessarily omits some of the mechanisms necessary to capture the business cycle fluctuations in construction, even conditional on the time path of prices. For example, the relationship between the price of construction and demand will be affected by the stock of existing structures. As explained by Rognlie et al. (2018), economic booms often involve overbuilding of homes, followed by "investment hangovers" where the demand for structures is satisfied by the existing homes, rather than new construction. Furthermore, many people view structures as a financial investment, implying that the demand for new construction depends on expectations of future asset prices, which are not included in the model (Guerrieri and Uhlig, 2016).

The climate damage from heat stress primarily affects the economy through the construction sector because the share of construction in investment value added is much larger than the shares of agriculture and mining in either investment or consumption value added. The elasticity of substitution between construction and the other intermediates, σ_x, is important for predicting how the share of construction value added will evolve over time, and hence how the climate damage from heat stress will evolve. Our estimated value of σ_x is near unity, implying that the share of investment value added from construction will be relatively constant over time, as it has been historically (see Figure 5). If instead, the substitution elasticity were near zero, as is true for the other substitution elasticities, then the low productivity growth in construction would cause construction to eventually take over the economy. This alternate outcome would amplify the difference in climate damage to consumption and investment productivity and make accounting for heterogeneous damages even more important.

We determine the sector-specific productivity growth rates from the historical changes in the relative prices. Relative prices in the model are inversely related to relative productivity inclusive of climate damage. The price in sector j relative to

[12] See Koh et al. (2020) for related data on the construction share of investment expenditure, rather than value added.

Figure 4: Model Fit: Consumption



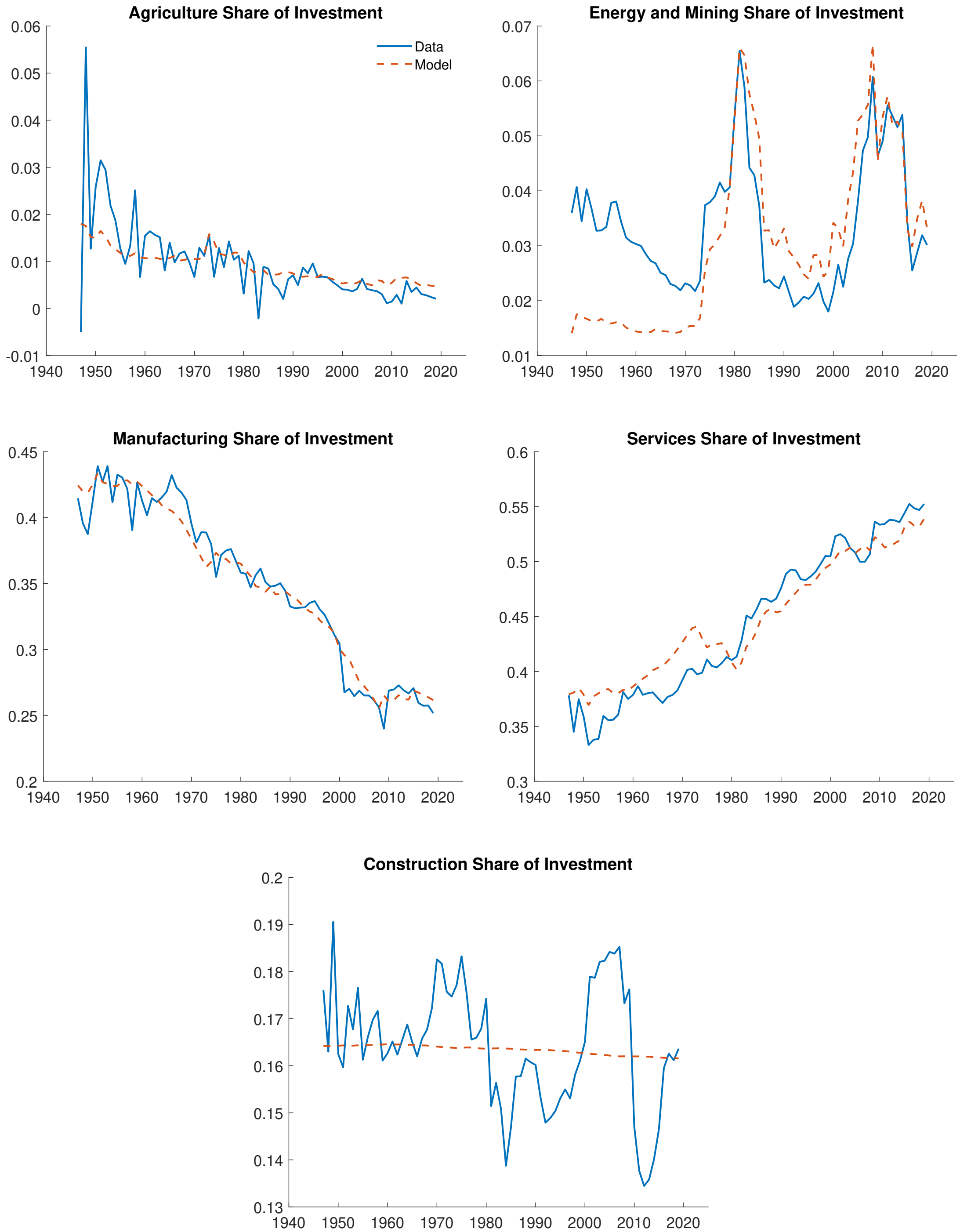
Note: the four panels plot the model (solid blue line) and empirical (dashed orange line) sector shares of consumption value added for agriculture, mining, manufacturing, and services sectors in the U.S. economy from 1947-2019.

the price of investment (the numeraire) is given by:

$$p_{jt} = \frac{(D_X(T_t)A_{Xt})^{1-\theta}}{(D_j(T_t)A_{jt})^{1-\theta}}. \tag{26}$$

Aggregating across sectors implies that $D_X(T_t)A_{Xt}$ equals total factor productivity (Y_t/K_t^{θ}) in the U.S. economy (see Appendix Section A.1.2). We calculate the time series for $D_X(T_t)A_{Xt}$ from data on aggregate capital and output per capita, deflated by the investment price deflator. Using the relationship in equation (26), the time series for $D_X(T_t)A_{Xt}$, and data on relative prices in each sector, we construct the

Figure 5: Model Fit: Investment



Note: the five panels plot the model (solid blue line) and empirical (dashed orange line) sector shares of investment value added for agriculture, mining, manufacturing, services and construction sectors in the U.S. economy from 1947-2019.

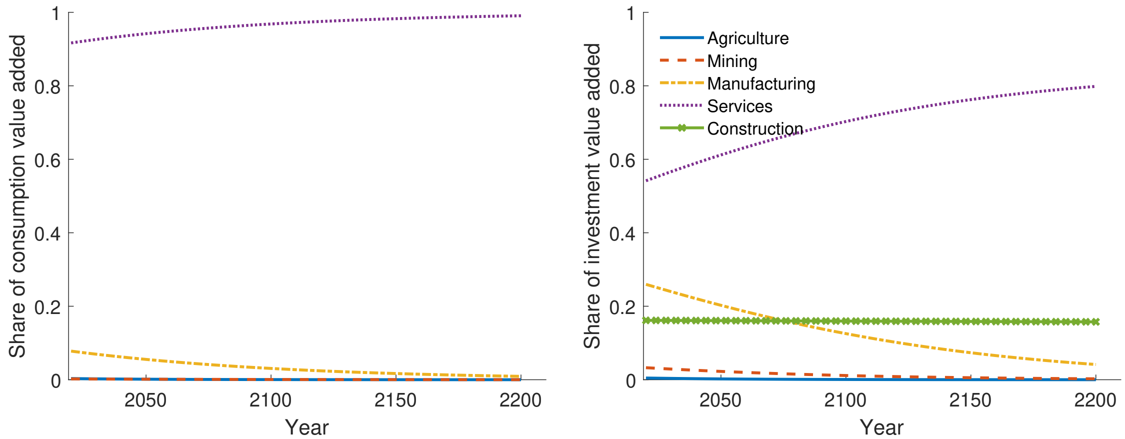
historical time series for $D_j(T_t)A_{jt}$ in each sector j. We use gridded weather data on daily average temperature from Schlenker and Roberts (2009) and information on relative humidity, solar irradiance, and wind speed to calculate the historical time series for D_j in each sector j (see Appendix Section B.3). Dividing the time series for $D_j(T_t)A_{jt}$ by the time series for $D_j(T_t)$ yields a time series for A_{jt}. For each sector j, we set the growth rate of productivity equal to the average growth rate of A_{jt} in the historical data. Construction and services have the lowest productivity growth rates, equal to -0.4 and 1.2 percent, respectively. The negative growth rate suggests that construction productivity is decreasing over time. This could reflect changes in regulations or increases in materials prices, among other possibilities. The productivity growth in agriculture is the highest at 6.7 percent. Mining and manufacturing are in the middle, with productivity growth rates of 4.4 and 3.6 percent, respectively.

We set θ equal to 0.33 to match the capital share of income, and set the inverse of the intertemporal elasticity of substitution, χ, equal to 1.5 as in Barrage (2020) and Nordhaus (2017). We choose β to target a real return on capital of 4 percent (McGrattan and Prescott, 2003). The relationship between β and the return on capital depends on the growth rates of the output per capita and the relative price of capital. Assuming that the U.S. economy since year 2000 is well approximated by a steady state, we use the average growth rates over this period. These values yield $\beta = 0.979$. We use this value of β to solve the dynamic model. We compute welfare using this market-based β and also using higher values of β to reflect a social planner that places more weight on future outcomes. We set the depreciation rate equal to 0.065, the average depreciation rate for fixed assets and consumer durables (calculated from NIPA Tables 1.1. and 1.3) over the period 2000-2019.

5.4 Sector shares

Figure 1 shows that investment in 2019 is more vulnerable to climate damage from heat stress, because a greater fraction of investment value added is produced outdoors. To understand the full impact of future changes in climate, it is important to consider how the sectoral composition of consumption and investment will evolve over time. Figure 6 plots the evolution of the sector shares of consumption (panel a) and investment (panel b) value added from 2020-2200 in the calibrated model,

Figure 6: Sector Shares of Consumption and Investment Value Added



(a) Consumption (b) Investment

Note: Panels (a) and (b) plot the sector shares of consumption and investment value added, respectively, for the model simulation without climate change.

under the assumption that climate remains constant at its 2019 level. The shares of consumption and investment value added from agriculture and mining are relatively small, less than 5 percent in 2020, and they fall further over time. In contrast, the construction share of investment value added is much larger, equal to almost 20 percent, and is relatively constant, consistent with the historical trends presented in Figure 5. These dynamics suggest the macroeconomic impacts of climate damage from heat stress primarily stem from effects on construction productivity.

6 Quantitative Results

The analytic model in Section 3 demonstrated that the split between consumption and investment productivity matters for how climate change impacts the economy. We now use our richer model to explore the quantitative implications of heterogeneous damages, focusing on the labor productivity losses caused by heat stress in the United States.

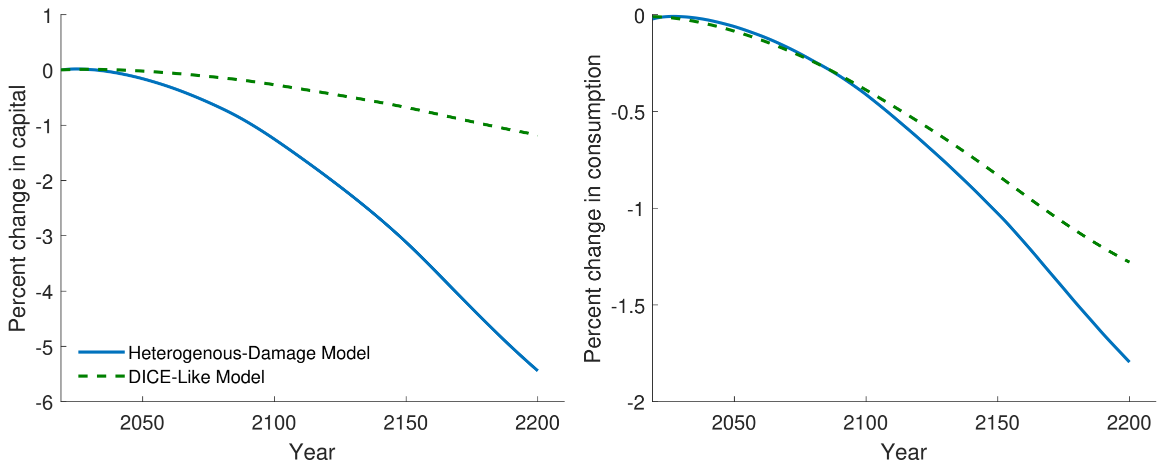
6.1 Core analysis

Our model with sector-specific damages and structural change allows climate change to differentially affect consumption and investment productivity. In contrast, many macro climate-economy models follow DICE and assume that climate change has equal impacts on consumption and investment productivity (e.g., Nordhaus and Boyer, 2003; Golosov et al., 2014; Barrage, 2020; Hassler et al., 2021). To quantify the importance of heterogeneous climate damages, we compare the macro impacts of climate change from two different model simulations. The first is our primary model described in Section 4. We refer to this model as the heterogeneous-damage (HD) model. The second is a DICE-like model in which we replace the sector-specific damage functions from our HD model with an aggregate damage function. The aggregate damage function has the same impact on every sector, meaning that it does not alter the relative productivity of consumption and investment. Following the standard approach in the macro climate-economy literature, we construct the aggregate damage function as a weighted average of the sector-specific damage functions (Nordhaus and Boyer, 2003; Barrage, 2020). The weights in each period are equal to the sector shares of GDP from a "no-climate-change" simulation of the HD model simulation, which is discussed in greater detail below. All other components of the DICE-like model are the same as the HD model.

To quantify the effects of climate change in the HD and DICE-like models, we compute a "climate-change" simulation of each model. In the climate-change simulation of the HD model, we feed in the sector-specific damage projections calculated in Section 5.2. In the climate-change simulation of the DICE-like model, we feed in the aggregate damage function derived from the weighted average of the sector-specific climate damages. We compare the outcomes from each climate-change simulation to a "no-climate-change" simulation that holds climate constant at its 2019 level. Since the HD and DICE-like models only differ in their damage specifications, the "no-climate-change" simulation is the same in both cases.

Panel (a) of Figure 7 plots the effect of climate change on capital in the HD model (solid blue line) and in the DICE-like model (dashed green line). Since heat stress has a greater impact on investment productivity, damages to investment productivity are higher in the HD model than in the DICE-like model. As a result, allowing

Figure 7: Effect of Climate Change on Capital and Consumption



(a) Capital (b) Consumption

Note: The panels show the effects of climate change on capital (panel a) and consumption (panel b) in the HD (solid blue line) and DICE-like (dashed green line) models. To incorporate climate change, we take the temperature realizations for the mean of the climate model predictions for RCP 8.5.

for heterogeneous damage from heat stress magnifies the effect of climate change on capital accumulation. By 2100, climate change reduces capital by 1.25 percent in the HD model but by less than 0.27 percent in the DICE-like model. By 2200, climate change reduces capital by over 5 percent in the HD model, compared to only 1.2 percent in the DICE-like model.

Allowing for heterogeneous damages also affects the decrease in consumption caused by climate change. Panel (b) of Figure 7 plots the effect of climate change on consumption in the HD model (solid blue line) and in the DICE-like model (dashed green line). Comparing the blue and the green lines reveals that the DICE-like model overstates the decrease in consumption in the near term (the green line is below the blue line prior to 2085) and understates the fall in consumption in the long term (the green line is above the blue line after 2085). The intuition for this result comes from the simple model presented in Section 3. Holding the size of the economy and the savings rate constant, only damages to consumption productivity have an immediate effect on the level of consumption. Damages to investment productivity slow the process of capital accumulation and decrease consumption in the long run.[13] Again, since heat stress has a greater impact on investment productivity, damages to investment productivity are higher in the HD model than in the DICE-like model, and damages to consumption productivity are lower in the HD model. Thus, short-run consumption losses are larger in the DICE-like model, but long-run consumption losses are larger in the HD model.

Lastly, we show that allowing for heterogeneous damages has important implications for the welfare costs of climate change. Following Barrage (2020), we use two measures of the consumption equivalent variation (CEV) to compare the welfare costs of climate change between the HD and DICE-like models. The permanent-change-in-consumption (PCC) CEV measures the percent increase in consumption a household would need in every period in the no-climate-change economy so that it is indifferent between living in the no-climate-change economy and the climate-change economy. The aggregate-initial-consumption (AIC) CEV measures the level change in year 2019 consumption that the household would need in the no-climate-change economy so that she is indifferent between living in the no-climate-change economy and the climate-change economy. Negative CEVs indicate that climate change makes households worse off. The first two rows in the first panel of Table 2 report the PCC CEV in the HD and DICE-like models and the first two rows in the second panel report the AIC CEV. The third row in each panel reports the ratio of the DICE-like CEV to the HD CEV. Importantly, these measures of the welfare cost only capture the climate damage from the labor-productivity losses caused by heat stress to the U.S. economy. They do not capture the full welfare cost of climate change, which would include all types of climate damage to all countries.

The welfare cost of climate change depends on the discount factor (β) used to weight consumption at different points in time. Since the HD and DICE-like models have different time profiles of consumption losses, the discount factor also matters for the relative welfare costs in the two models. The first column of Table 2 corresponds to a case where the social planner calculates the welfare cost using the same discount factor as market participants. Individuals place less weight on future utility, which

[13] Climate change leads to a slight increase in the savings rate, with a larger increase in the HD model. As a result, climate change actually increases consumption for the first five years in the HD model. This increase comes at the expense of capital accumulation and future consumption.

is captured by the fact that $\beta < 1$.[14] Nordhaus (2007) argues that climate policy should be designed using a discount factor that is consistent with market behavior. In a recent survey, however, Drupp et al. (2018) found that many economists favor normative frameworks that place higher weight on future outcomes, when compared to market participants. The modal response was that policy analysis should place equal moral weight on outcomes at all times ($\beta = 1$), a result consistent with the arguments of Stern et al. (2006).[15] The remaining columns of Table 2 report the welfare costs of climate change calculated with higher discount factors capturing the perspective of a social planner that places a greater moral weight on future outcomes.[16]

Referring to the third row of either panel of Table 2, the ratio is less than one for all values of β, implying that the DICE-like model underestimates the welfare cost of climate change. Moving to the right across the columns of Table 2 shows that the ratio falls as the β rises. The market-based level of $\beta = 0.9788$ implies that the DICE-like model understates the welfare cost of climate change by 4 percent. If instead the social planner puts almost equal moral weight on consumption at all times, i.e., $\beta = 0.995$, then the DICE-like model understates the welfare cost of climate change by 24 percent using the PCC CEV and by 18 percent using the AIC CEV. These results reflect the fact that the HD model predicts larger losses in consumption in the long run. When β is larger, these losses generate higher welfare costs from today's perspective.

Ultimately, the differences between the HD and DICE-like models in Figure 7 demonstrate that how damages are split between consumption and investment productivity has important quantitative implications for the impact of climate change on the economy. Using an aggregate damage function understates the fall in capital, overstates the short-run fall in consumption, and understates the long-run fall in consumption. These different dynamics have important implications for the welfare

[14] The parameter β captures the pure time preference. Even with $\beta = 1$, individuals discount future reductions in consumption, because of diminishing marginal returns in the utility function. The literature on discounting often focuses on rates of time preference, rather than discount factors. The two are related by $\beta = (1 + \rho)^{-1}$, where ρ is the pure rate of time preference. We focus on the discount factor to be consistent with the macro literature.

[15] In our applications, we use values of β that are strictly less than one, so that the finite time horizon does not affect the results.

[16] Barrage (2018) demonstrates that high social discount factors have important implications for optimal policy. Here, we focus only on the costs of climate under a 'business as usual' (BAU) case.

Table 2: Welfare Costs of Climate Damage From Heat Stress in the U.S.

	Discount Factor (β)				
	0.979	0.980	0.985	0.990	0.995
Permanent-Change-in-Consumption CEV (percent)					
DICE-like model	-0.148	-0.157	-0.213	-0.304	-0.462
Heterogeneous-Damage model	-0.154	-0.167	-0.241	-0.369	-0.606
Ratio	0.96	0.94	0.88	0.82	0.76
Aggregate-Initial-Consumption CEV (billions of 2019 dollars)					
DICE-like model	-739	-818	-1326	-2328	-4440
Heterogeneous-Damage model	-772	-865	-1487	-2755	-5419
Ratio	0.96	0.95	0.89	0.84	0.82

Note: This table compares the welfare costs of the climate damage from heat stress in the HD and DICE-like models for different values of the discount factor. The first panel reports the welfare cost measured by the permanent-change-in-consumption CEV and the second panel reports the welfare cost measured by the aggregate-initial-consumption CEV, where CEV is the consumption equivalent variation.

consequences of climate change. The analysis in Table 2 reveals that accounting for heterogeneous damages meaningfully increases the welfare consequences of the climate damage from heat stress, especially when using a normative framework that weights current and future outcomes similarly. Additionally, the DICE-like model substantially underestimates the effect of changes in β on the welfare costs of heat stress, suggesting that the discount factor could be an even more important component of the welfare costs of climate change than has been previously realized (e.g., Dietz and Stern, 2008; Sterner and Persson, 2008; Drupp and Hänsel, 2018).

6.2 Tail risk

Our core analysis focuses on the climate damage from heat stress for the mean predictions for future temperature change under RCP 8.5. Many researchers have emphasized that there is substantial uncertainty over future climate trajectories, even given a path of the carbon concentration (e.g., Heal and Millner, 2014). Well-designed policy must take into account the full distribution of potential outcomes. Indeed, some scholars argue that climate policy should be thought of as an insurance policy against the worst possible outcomes (e.g., Weitzman, 2009; Heal, 2017).

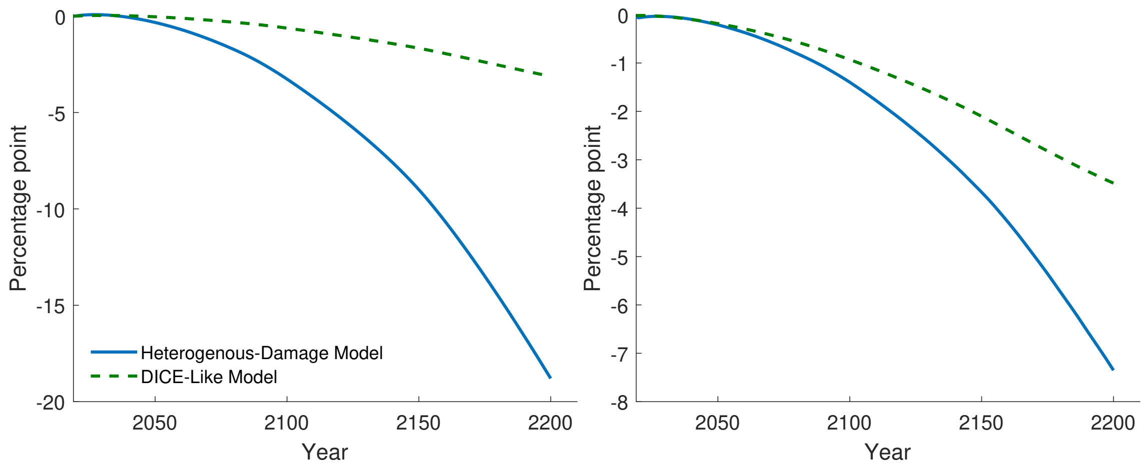
We simulate the effect of climate change in the HD and DICE-like models for a realization of temperature from the right tail of the distribution. We take the temperature projection from the model surrogate, scaled GFDL CM3, which is in the 95th percentile of the distribution reported in Rasmussen et al. (2016). We follow the same procedure as before to calculate national damages for outdoor sectors with this more extreme path of temperature realizations.

Figure 8 plots the effect of climate change on capital (panel a) and consumption (panel b) in the HD (solid blue line) and DICE-like (dashed green line) models. As one would expect, climate change leads to larger reductions in capital and consumption for the tail temperature realization compared to the effects from the mean realization, plotted in Figure 7. More interestingly, however, the tail realization of temperature magnifies the differences between the DICE-like and HD models. For example, the difference between the blue and green lines in 2200 reveals that the DICE-like model under-predicts the fall in capital by over 16 percentage points under the tail realization of temperature (panel (a) of Figure 8) , compared to only 4 percentage points under the mean realization (panel (a) of Figure 7) . As a result of the larger decrease in capital under the tail realization, consumption losses in the HD model exceed those in the DICE-like model sooner. The switch occurs in 2041, compared to 2085 under the mean temperature projection. Thus, if climate damage is more severe, then the magnitude of the errors from abstracting from heterogeneous damages increases, causing the DICE-like model to under-predict consumption losses at closer time horizons.

6.3 Comparison to other policy-relevant models

Thus far, we have focused on comparing our results to the DICE model, which is the most prominent climate-economy model. Our results are also immediately relevant to a wider set of computable general equilibrium models used for policymaking. In the U.S., the Interagency Working Group on Greenhouse Gas Emissions (IWG) uses three models to predict the welfare consequences of climate change: DICE, FUND and PAGE (IWG, 2016). As explained above, DICE assumes equal damage to consumption and investment. FUND and PAGE implicitly assume that all damages occur to consumption (Hope, 2006; Anthoff and Tol, 2014). More specifically, they

Figure 8: Effects of Climate Change: High Temperature Realization



(a) Capital (b) Consumption

Note: The panels show the effects of climate change on capital (panel a) and consumption (panel b) in the HD (solid blue line) and DICE-like (dashed green line) models. To incorporate climate change, we take the temperature realizations from the 95th percentile of the climate model predictions for RCP 8.5.

assume that GDP follows an exogenous path in the absence of climate damages. Changes in climate perturb output from this path within a period, but have no effect on future levels of GDP, implying no effect on capital. Given that these models do not have endogenous consumption decisions, it is not possible to do a meaningful welfare comparison with our model. Intuitively, however, the impact of accounting for heterogeneous damages in these models will be similar to the impacts in DICE. As shown in Figure 7, the DICE-like model predicts only small changes in the capital stock. FUND and PAGE make the slightly more extreme prediction that there is no change in capital stock, implying that the differences we observe would be slightly larger if we compared the HD model to FUND or PAGE instead of to DICE.

7 Conclusion

Our work is part of a broader effort to understand how climate damage will impact the economy. A micro literature focuses on carefully identifying the effects of climate change on productivity in particular sectors and geographic regions (Auffhammer, 2018). An emerging macro literature focuses on how to incorporate these estimates into general equilibrium models. So far, this literature has studied the equilibrium consequences of these sector-level damages across space (e.g., Desmet and Rossi-Hansberg, 2015; Costinot et al., 2016; Nath, 2020). Our paper instead focuses on the general equilibrium consequences of the sector-level damages for the dynamics of consumption and investment.

We connect our analysis to the broader macro climate-economy literature that uses aggregate damage functions to abstract from heterogeneous damages. In particular, we find that aggregate damage functions will overestimate short-run consumption losses and underestimate long-run consumption losses when investment is more vulnerable to climate change. The opposite implications hold if instead consumption is more vulnerable. We quantify these effects for the climate damage from heat stress and find that accounting for heterogeneous damages increases the welfare cost of climate change by 4 to 24 percent, depending on the discount factor.

While our quantitative analysis focuses on the labor productivity losses caused by heat stress, the same mechanisms apply more broadly to other types of climate damage. Since consumption and investment have different sectoral compositions, any type of climate damage that varies across sectors will generate different damage to consumption and investment productivity. For example, manufacturing production may be less likely to occur along coastlines, where property values are high, making it less susceptible to climate damage from sea level rise. Our results suggest that understanding the heterogeneous sector-level impacts of other types of climate damage is an important step in the process of quantifying the full dynamic consequences of climate change.

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Online Appendix

A Solution to the Quantitative Model

We use the techniques developed in Herrendorf et al. (2013, 2014, 2021); Garcia-Santana et al. (2021) to solve the quantitative model. Section A.1 describes the equilibrium. Section A.2 derives the aggregate consumption price index and aggregate consumption expenditure.

A.1 Description of the Equilibrium

A.1.1 Equilibrium Prices

Production factors are perfectly mobile across sectors, implying that the wage and rental rates will be equalized. Factor prices for production sector j are

$$w_t = p_{jt}(1-\theta)\left(\frac{k_{jt}}{n_{jt}}\right)^{\theta}\left(D_j(T_t)A_{jt}\right)^{1-\theta}, \tag{A.1}$$

$$r_t = p_{jt}\theta\left(\frac{k_{jt}}{n_{jt}}\right)^{\theta-1}\left(D_j(T_t)A_{jt}\right)^{1-\theta}, \tag{A.2}$$

where w_t is the wage, r_t is the rental rate on capital, and p_{jt} is the price of output from sector j. Taking the ratio of factor prices,

$$\frac{w_t}{r_t} = \frac{1-\theta}{\theta}\frac{k_{jt}}{n_{jt}}. \tag{A.3}$$

With factor mobility, the left-hand side does not vary across sectors. By assumption, θ is also common across sectors. This implies that the capital-labor ratio is the same in all sectors and is therefore equal to the aggregate capital-labor ratio.

In addition, perfect competition implies that

$$p_{jt} = \left(D_{jt}A_{jt}\right)^{\theta-1}\left(\frac{w_t}{1-\theta}\right)^{1-\theta}\left(\frac{r_t}{\theta}\right)^{\theta}. \tag{A.4}$$

From equations (13) and (14), profit maximization for the investment aggregator yields

$$1 = \left(\xi_b p_{bt}^{1-\sigma_x} + \xi_z p_{zt}^{1-\sigma_x}\right)^{\frac{1}{1-\sigma_x}}, \tag{A.5}$$

where

$$p_{zt} = \left(\sum_{j \neq b} \xi_z p_{x_j t}^{1-\sigma_z}\right)^{\frac{1}{1-\sigma_z}}. \tag{A.6}$$

Combined with equation (A.4), this yields

$$\begin{aligned} 1 = &\left[\left(\frac{w_t}{1-\theta}\right)^{1-\theta}\left(\frac{r_t}{\theta}\right)^{\theta}\right] \\ &\cdot \left[\xi_b \left(D_b(T_t)A_{bt}\right)^{(\theta-1)(1-\sigma_x)} + \xi_z \left(\sum_{j \neq b} \xi_j \left(\left(D_j(T_t)A_{jt}\right)^{(\theta-1)(1-\sigma_z)}\right)^{\frac{1}{1-\sigma_z}}\right)^{1-\sigma_x}\right]^{\frac{1}{1-\sigma_x}}. \end{aligned} \tag{A.7}$$

For expositional simplicity, we define

$$\begin{aligned} &\left(D_X(T_t)A_{Xt}\right)^{\theta-1} \\ \equiv &\left[\xi_b \left(D_b(T_t)A_{bt}\right)^{(\theta-1)(1-\sigma_x)} + \xi_z \left(\sum_{j \neq b} \xi_j \left(\left(D_j(T_t)A_{jt}\right)^{(\theta-1)(1-\sigma_z)}\right)^{\frac{1}{1-\sigma_z}}\right)^{1-\sigma_x}\right]^{\frac{1}{1-\sigma_x}}, \end{aligned} \tag{A.8}$$

which captures the overall productivity of investment after taking climate impacts into account. There is no closed form way to separate A_{Xt} and $D_X(T_t)$. Since production is perfectly competitive, the investment aggregator makes zero profits and

$$X_t = \sum_j p_{jt} x_{jt}. \tag{A.9}$$

Now, we take equation (A.4) for any production sector j and divide by equation (A.7). This yields

$$p_{jt} = \frac{\left(D_X(T_t)A_{Xt}\right)^{1-\theta}}{\left(D_j(T_t)A_{jt}\right)^{1-\theta}}. \tag{A.10}$$

Because of the symmetry between sectors, the relative price of the good from any sector depends only on the relative efficiency of production, which in turn depends on technology and exposure to climate. Since the aggregate investment good is the numeraire, the price of output from any sector is the inverse of productivity relative to investment-specific productivity. This implies that all relative prices in the model are independent of the household decisions. This fact greatly simplifies the computational solution.

A.1.2 Gross Domestic Product (GDP)

GDP in this economy is given by

$$Y_t = w_t + r_t K_t = X_t + \sum_{j \neq b} p_{jt} c_{jt} = \sum_j p_{jt} y_{jt} = K_t^{\theta} \left(A_{Xt} D_X(T_t) \right)^{1-\theta}. \tag{A.11}$$

The first equality expresses GDP in terms of income. The second equality splits GDP between value added in consumption and investment, and the third equality uses equations (12) and (A.9) to express GDP in terms of value added in each of the underlying production sectors. The final equality uses equations (15), (16), and (A.10) to express output in terms of aggregate capital and productivity in the investment sector. The last equality holds because we take the aggregate investment good as the numeraire.

A.1.3 Household Problem

Using the utility function equations, (18) and (19), and the market clearing condition for the final good, equation (A.11), the full problem of the representative household is given by

$$\max_{\{c_{at}, c_{mt}, c_{et}, c_{st}, K_{t+1}\}_{t=0}^{T}} (1-\chi)^{-1} \sum_{t=0}^{t_{\max}} \beta^t \left(\sum_{j \neq b} \omega_j^{\frac{1}{\sigma_c}} (c_{jt} + \bar{c}_j)^{\frac{\sigma_c - 1}{\sigma_c}} \right)^{\frac{\sigma_c(1-\chi)}{(\sigma_c - 1)}} \tag{A.12}$$

$$\text{s.t.} \sum_{j \neq b} p_{jt} c_{jt} + K_{t+1} = (1 - \delta + r_t) K_t + w_t, \tag{A.13}$$

$$(1-\delta) K_t \leq K_{t+1}, \tag{A.14}$$

$$c_{j,t} \geq 0, \tag{A.15}$$

where equation (A.14) reflects the non-negativity of investment ($X_t \geq 0$). Existing capital cannot be transformed into a consumption good. In the quantitative application, the investment non-negativity constraint will only bind as t approaches $t_{\max}$, which occurs well beyond the time period on which we focus. We also find that changing $t_{\max}$ has no impact on the outcomes we study. Consumption of each good is always strictly positive in our quantitative application, and we ignore equation (A.15) in everything that follows.

Let $\beta^t \lambda_t$ and $\beta^t \mu_t$ be the Lagrange multipliers on the period t budget and investment non-negativity constraints, respectively. Noting the definition in equation (18), the first order conditions can be written as

$$c_{jt}\ (\forall j, t): \qquad C_t^{-\chi} \omega_j^{\frac{1}{\sigma_c}} \left(c_{jt} + \bar{c}_j\right)^{\frac{-1}{\sigma_c}} C_t^{\frac{1}{\sigma_c}} = \lambda_t p_{jt}, \tag{A.16}$$

$$K_{t+1}\ (t < t_{\max}): \qquad \lambda_t - \mu_t = \beta \lambda_{t+1}(1 - \delta + r_{t+1}) - \beta \mu_{t+1}(1 - \delta), \tag{A.17}$$

$$K_{t_{\max}+1}: \qquad \lambda_{t_{max}} = \mu_{t_{max}}, \tag{A.18}$$

$$X_t \geq 0\ (\forall t): \qquad \mu_t \left[(1 - \delta) K_t - K_{t+1}\right] = 0. \tag{A.19}$$

In Appendix Section A.2.1, we show that combining equation (A.16) for each j yields

$$C_t^{-\chi} = \lambda_t \left[\sum_{j \neq b} \omega_j p_{jt}^{1-\sigma_c}\right]^{\frac{1}{1-\sigma_c}}. \tag{A.20}$$

Herrendorf et al. (2014) derive this result with $\chi = 1$. Noting that $C_t^{-\chi}$ is the marginal utility of aggregate consumption and λ_t is the (current-value) shadow value of income, we refer to

$$P_t \equiv \left[\sum_{j \neq b} \omega_j p_{jt}^{1-\sigma_c}\right]^{\frac{1}{1-\sigma_c}} \tag{A.21}$$

as the consumption price index.

Using equations (A.17), (A.19), (A.20), and (A.21), the Euler equation is

$$\left(\frac{C_{t+1}}{C_t}\right)^{\chi} = \beta \left(1 - \delta + r_{t+1}\right) \frac{P_t}{P_{t+1}}, \tag{A.22}$$

for consecutive periods t and $t+1$ where the investment non-negativity constraint does not bind. More generally, for any $t < t_{\max}$,

$$C_t^{-\chi} P_t^{-1} - \mu_t = \beta C_{t+1}^{-\chi} P_{t+1}^{-1} (1 - \delta + r_{t+1}) - \beta \mu_{t+1} (1 - \delta). \tag{A.23}$$

The non-negativity constraint will bind in the final period, yielding the terminal condition

$$K_{t_{\max}+1} = (1 - \delta) K_{t_{\max}}. \tag{A.24}$$

To understand the role of the non-negativity constraint, it helps to consider the case where the constraint only binds in the final period. In this case, the Euler equation for the penultimate period is

$$\left(\frac{C_{t_{\max}}}{C_{t_{\max}-1}} \right)^{\chi} = \beta r_{t_{\max}} \frac{P_{t_{\max}-1}}{P_{t_{\max}}}.$$

This is similar to the unconstrained case, except that the household gets no value from the un-depreciated portion, $(1 - \delta)$, of capital, which cannot be transformed into consumption.

A.1.4 Aggregate Dynamics

In this section, we show how to find the dynamics of C_t and K_{t+1} independently of the sector-level allocations. Once again, the analysis closely follows Herrendorf et al. (2014). To find the rental rate, combine equation (A.2) for any sector j with equation (A.10), which yields

$$r_t = \theta K_t^{\theta-1} \left(A_{Xt} D_X(T_t) \right)^{1-\theta}. \tag{A.25}$$

The Euler equation becomes

$$C_t^{-\chi} = \beta \left(1 - \delta + \theta K_{t+1}^{\theta-1} \left(A_{Xt+1} D_X(T_{t+1}) \right)^{1-\theta} \right) \frac{P_t}{P_{t+1}} C_{t+1}^{-\chi}, \tag{EE}$$

for any two periods t and $t+1$ where the non-negativity constraint does not bind, and

$$C_t^{-\chi} - \mu_t = \beta \left(1 - \delta + \theta K_{t+1}^{\theta-1} \left(A_{Xt+1} D_X(T_{t+1}) \right)^{1-\theta} \right) \frac{P_t}{P_{t+1}} C_{t+1}^{-\chi} - \beta \mu_{t+1} (1 - \delta), \tag{EE-Cons}$$

more generally.

Now, we turn to deriving a law of motion for capital. Appendix Section A.2.2 derives the following result,

$$\sum_{j\neq b} p_{jt}c_{jt} = P_t C_t - \sum_{j\neq b} p_{jt}\bar{c}_j. \tag{A.26}$$

Herrendorf et al. (2014) derive an identical result with $\chi = 1$. This equation expresses total expenditure on consumption goods in terms of aggregate consumption and variables exogenous to household decision-making. Using equation (A.11), the law of motion for capital, equation (17), can now be re-written as

$$K_{t+1} = \underbrace{K_t^{\theta}\left(A_{Xt}D_X(T_t)\right)^{1-\theta}}_{Y_t} - \underbrace{\left(P_t C_t - \sum_{j\neq b} p_{jt}\bar{c}_j\right)}_{\text{Consumption Expenditure}} + (1-\delta)K_t. \tag{LOM}$$

Combined with equation (A.24), this yields the terminal condition

$$P_{t_{\max}} C_{t_{\max}} = K_{t_{\max}}^{\theta}\left(A_{Xt_{\max}}D_X(T_{t_{\max}})\right)^{1-\theta} + \sum_{j\neq b} p_{jt_{\max}}\bar{c}_j. \tag{TC}$$

Recall from equation (A.10) that prices are determined by technology and can be found independently of all other variables. In this section, we have now written the key dynamic equations, (EE-Cons) and (LOM), and boundary conditions, K_0 and (TC), in terms of aggregate variables only. Together with the complementary slackness condition (A.19), they can be used to find the aggregate allocations independently of the sector-level allocations. Specifically, taking the initial capital stock and sequence of prices and technology levels as given, the aggregate allocations are the solution to the problem of a social planner that chooses $\{C_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize (19), subject to the aggregate budget constraint (LOM) and the investment non-negativity condition (A.14).

A.1.5 Static Allocations

It is straightforward to find the sector-level, static allocations after solving for the sequence of prices and the aggregate dynamics. To start, we consider the quantity of investment goods produced. From equations (A.11) and (A.26),

$$X_t = (A_{Xt} D_X(T_t))^{1-\theta} K_t^{\theta} - P_t C_t + \sum_{j \neq b} p_{jt} \bar{c}_j, \tag{A.27}$$

which can be taken as given when finding the sector-level allocations. From equations (13) and (14), the first order conditions of the investment good aggregator can be written as

$$\begin{aligned} x_{bt} &= \xi_b X_t p_{bt}^{-\sigma_x}, && \text{(A.28)} \\ x_{zt} &= \xi_z X_t p_{zt}^{-\sigma_x}, && \text{(A.29)} \\ x_{jt} &= \xi_j x_{zt} p_{jt}^{-\sigma_z} p_{zt}^{\sigma_x}, \quad j \neq b. && \text{(A.30)} \end{aligned}$$

Conditional on X_t and relative prices, these equations can be solved in order, determining x_{jt} for all j. In addition, we can use these equations and (A.6) to derive expressions for sector shares of investment expenditure, which are important for the calibration:

$$\begin{aligned} \frac{p_{b,t} x_{b,t}}{X_t} &= \xi_b p_{bt}^{1-\sigma_x}, && \text{(A.31)} \\ \frac{p_{j,t} x_{j,t}}{X_t} &= \xi_z \xi_j p_{jt}^{1-\sigma_j} \left(\sum_{j \neq b} \xi_z p_{x_j t}^{1-\sigma_z} \right)^{\frac{\sigma_z - \sigma_x}{1-\sigma_z}}, \ j \neq b. && \text{(A.32)} \end{aligned}$$

To find consumption allocations for each $j \neq b$, we combine equation (A.16), (A.20), and (A.21) to arrive at:

$$c_{jt} = \omega_j \left(\frac{p_{jt}}{P_t} \right)^{-\sigma_c} C_t - \bar{c}_j, \tag{A.33}$$

where c_{jt} is the only unknown. In addition, applying equations (A.21) and (A.26), gives the sector share of consumption as

$$\frac{p_{jt} c_{jt}}{\sum_{j \neq b} p_{j,t} c_{j,t}} = \frac{\omega_j p_{j,t}^{1-\sigma_c}}{\sum_{j \neq b} \omega_j p_{jt}^{1-\sigma_c}} \cdot \left(1 + \frac{\sum_{j \neq b} p_{j,t} \bar{c}_{j,t}}{\sum_{j \neq b} p_{j,t} c_{j,t}} \right) - \frac{p_{j,t} \bar{c}_j}{\sum_{j \neq b} p_{j,t} c_{j,t}} \tag{A.34}$$

With the equilibrium quantities of consumption and investment, it is straightforward to find total value added and the quantity of labor inputs in each underlying production sector. To do so, we use the production function and market clearing condition for each sector, equations (10) and (12):

$$c_{jt} + x_{jt} = y_{jt} = K_t^{\theta} \left(D_j(T_t)A_{jt}\right)^{1-\theta} n_{jt}. \tag{A.35}$$

The second equation also uses the fact that K_{jt} is both the aggregate capital-labor ratio and the capital-labor ratio in each sector.

A.1.6 Numerical Solution

The separability of the dynamic and static allocations makes it easier to numerically solve the model. We simulate a finite horizon economy that ends in period $t_{\max}$. We use equations (A.8), (A.10), (A.21) to derive the time paths of investment productivity, sector-level prices, and the relative price of consumption (P_t), respectively, given time paths for the sector-specific productivity and climate damages. Conditional on these variables, we then solve the social planner problem described in Section A.1.4 recursively to find the the dynamic time paths of X_t and C_t. Finally, we use equations (A.28) – (A.30) and (A.33) to derive the time paths of sector-specific consumption and investment, respectively.

A.2 Additional Derivations

A.2.1 Derivation of Consumption Price Index

Here, we derive equation (A.20). To start, use equation (18) to re-write the first order condition for consumption of good $c_{j,t}$, equation (A.16), as

$$C_t^{-\chi} \omega_j^{\frac{1}{\sigma_c}} \left(c_{jt} + \bar{c}_j\right)^{\frac{-1}{\sigma_c}} \left[\sum_{j \neq b} \omega_j^{\frac{1}{\sigma_c}} \left(c_{jt} + \bar{c}_j\right)^{\frac{\sigma_c - 1}{\sigma_c}}\right]^{\frac{1}{\sigma_c - 1}} = \lambda_t p_{jt}. \tag{A.36}$$

Raising everything to the power of $(1 - \sigma_c)$ yields

$$C_t^{-\chi(1-\sigma_c)} \omega_j^{\frac{1-\sigma_c}{\sigma_c}} \left(c_{jt} + \bar{c}_j\right)^{\frac{-(1-\sigma_c)}{\sigma_c}} \left[\sum_{j \neq b} \omega_j^{\frac{1}{\sigma_c}} \left(c_{jt} + \bar{c}_j\right)^{\frac{\sigma_c - 1}{\sigma_c}}\right]^{-1} = \lambda_t^{1-\sigma_c} p_{jt}^{1-\sigma_c}. \tag{A.37}$$

Multiplying both sides by ω_j and simplifying exponents gives

$$C_t^{-\chi(1-\sigma_c)}\omega_a^{\frac{1}{\sigma_c}}\left(c_{jt}+\bar{c}_j\right)^{\frac{\sigma_c-1}{\sigma_c}}\left[\sum_{j\neq b}\omega_j^{\frac{1}{\sigma_c}}\left(c_{jt}+\bar{c}_j\right)^{\frac{\sigma_c-1}{\sigma_c}}\right]^{-1}=\lambda_t^{1-\sigma_c}\omega_j p_{jt}^{1-\sigma_c}. \tag{A.38}$$

Taking the sum over all $j \neq b$ yields

$$C_t^{-\chi(1-\sigma_c)}=\lambda_t^{1-\sigma_c}\left(\sum_{j\neq b}\omega_j p_{jt}^{1-\sigma_c}\right). \tag{A.39}$$

To arrive at equation (A.20), raise everything to the power of $1/(1-\sigma_c)$.

A.2.2 Derivation of Aggregate Consumption Expenditure

Here, we derive equation (A.26). We again start by considering equation (A.16). Note that $C_t^{\frac{1}{\sigma_c}}=\left[\sum_{j\neq b}\omega_j^{\frac{1}{\sigma_c}}\left(c_{jt}+\bar{c}_j\right)^{\frac{\sigma_c-1}{\sigma_c}}\right]^{-1}C_t$. Applying this result and multiplying both sides by $(c_{jt}+\bar{c}_j)$ yields

$$C_t^{1-\chi}\left[\sum_{j\neq b}\omega_j^{\frac{1}{\sigma_c}}\left(c_{jt}+\bar{c}_j\right)^{\frac{\sigma_c-1}{\sigma_c}}\right]^{-1}\omega_j^{\frac{1}{\sigma_c}}\left(c_{jt}+\bar{c}_j\right)^{\frac{\sigma_c-1}{\sigma_c}}=\lambda_t p_{jt}\left(c_{jt}+\bar{c}_j\right). \tag{A.40}$$

Taking the sum over all $j \neq b$ yields

$$C_t^{1-\chi}=\lambda_t\left[\sum_{j\neq b}p_{jt}\left(c_{jt}+\bar{c}_j\right)\right]. \tag{A.41}$$

From equations (A.20) and (A.21), $\lambda_t = P_t^{-1}C_t^{-\chi}$. Plugging in gives

$$P_tC_t=\sum_{j\neq b}p_{jt}\left(c_{jt}+\bar{c}_j\right). \tag{A.42}$$

To derive equation (A.26), distribute the right-hand side and bring all terms with $\bar{c}_j$ to the left-hand side.

B Data and Calibration

B.1 Consumption and Investment Value Added

We closely follow the procedures outlined by the BEA (Horowitz et al., 2006), Herrendorf et al. (2013), and Garcia-Santana et al. (2021) to calculate sector-level consumption and investment value added in the U.S. data from 1947-2019. There are two key assumptions embedded in this approach (Horowitz et al., 2006, p.1-3). First, the *principle of proportionality* says that the ratio of inputs to outputs remains constant over a wide range of output levels. Second, the *principle of homogeneity* says that each industry has a single production function, which holds regardless of how the output from that sector is used.

We use $z = 1, \ldots, Z$ to denote industries and $v = 1, \ldots, V$ to denote commodities. In addition, let i_Z denote the unit column vectors of length Z and i_V denote the unit column vector of length V. Similarly, let I_Z be the $(Z \times Z)$ identity and I_V be the $(V \times V)$ identity matrix. Superscript ^{T} denotes transpose and superscript $^{-1}$ denotes inverse.

The inputs into our calculation are the Make and Use tables from the BEA.[17] The Use table "shows the consumption of commodities by industries, as well as the commodity composition of gross domestic product (GDP) and the industry distribution of value added."[18] We split this table into three components. The first is a $(V \times Z)$ matrix $\mathbf{U}$ where element U_{vz} is the quantity of commodity v purchased by industry z as an intermediate input. The second is a matrix of final consumption expenditure on each commodity v. From this final consumption matrix, we aggregate across categories of consumption expenditure to form the $(V \times 1)$ vector e^C, which gives the total value of commodity v that is used in consumption. It includes consumption by households and government, as well as net imports. The third component of the Use table is a matrix of final investment expenditure on each commodity v. It includes public and private investment, as well as changes in inventories. From this final investment matrix, we form the $(V \times 1)$ vector e^X, which gives the total value

[17] The make and use tables are available from the BEA for three periods: 1947-1962, 1963-1996, and 1997-2019. The data are denominated in millions of U.S. dollars. https://www.bea.gov/industry/input-output-accounts-data

[18] https://www.bea.gov/help/glossary/use-table

of commodity v that is used in investment. The $(V \times 1)$ vector $e = e^C + e^X$ gives total final expenditure on each commodity. The Make table ($\mathbf{M}$) is a $(Z \times V)$ matrix, where element $\mathbf{M}_{zv}$ shows the production of commodity v by industry z. We use this table directly in our calculations.

To start, we need to deal with *scrap*, unintentional byproducts created by consuming or investing industry output. After consumption or investment has taken place, scrap can be sold as an input, but it does not count as part of gross output. The Make table classifies scrap as a commodity. Let h be the $(Z \times 1)$ vector where h_z is value of scrap created by using the output of industry z. We also define the $(Z \times 1)$ vector p, where p_z is the ratio of scrap production to total production in industry z. We define the $(Z \times V)$ matrix $\tilde{\mathbf{M}}$ as the $\mathbf{M}$ matrix after zeroing out the column corresponding to scrap.

Next, we construct several helpful matrices. We start be constructing a $(V \times 1)$ vector q, where q_v is the gross production of commodity q,

$$q = \left(i_Z^T \mathbf{M}\right)^T, \tag{B.1}$$

and a $(Z \times 1)$ vector g, where g_z is total output from industry z,

$$g = (\mathbf{M} i_V). \tag{B.2}$$

Both of these vectors include scrap. It is helpful to note the following two identities:

$$q = \mathbf{U} i_Z + e, \tag{B.3}$$

$$g = \tilde{\mathbf{M}} i_V + h. \tag{B.4}$$

The first identity breaks up total commodity production between commodities used as intermediate inputs ($\mathbf{U} i_Z$) and those devoted to final uses (e). The second identity splits production in each industry between non-scrap ($\tilde{\mathbf{M}} i_V$) and scrap (h).

Next, we create the $(Z \times Z)$ diagonal matrix $\hat{g}$, where the $\hat{g}_{zz} = g_z$ and then compute the $(V \times Z)$ matrix

$$\mathbf{B} = \mathbf{U} \hat{g}^{-1}, \tag{B.5}$$

where $\mathbf{B}$ is the *direct input coefficient* matrix. Element $\mathbf{B}_{vz}$ is the ratio of (intermediate spending on commodity v by industry z) to (total production of industry z). We also create $(V \times V)$ diagonal matrix $\hat{q}$, where the element $\hat{q}_{vv} = q_v$, and compute the $(Z \times V)$ matrix

$$\mathbf{D} = \tilde{\mathbf{M}}\hat{q}^{-1}, \tag{B.6}$$

where $\mathbf{D}$ is the *market share* matrix. Element $\mathbf{D}_{zv}$ is the (total production of commodity v by industry z) divided by (total expenditure on commodity v). Finally, we create $(Z \times Z)$ diagonal matrix $\hat{p}$, where $\hat{p}_{zz} = p_z$, and note the identity

$$h = \hat{p}g. \tag{B.7}$$

Our next goal is to compute the total requirements ($\mathbf{R}$) matrix, which links final expenditure on each commodity to gross production in each industry. More specifically, element $\mathbf{R}_{zv}$ shows the dollar value of industry z's production required (directly or indirectly) to produce one dollar of commodity v for final expenditure, accounting for input-output linkages. Recall that g_z is total production in sector g, and e_v is expenditure on commodity v for final uses. By definition, $\mathbf{R}$ is the $(Z \times Z)$ matrix that solves

$$g = \mathbf{R}e. \tag{B.8}$$

To derive an expression for $\mathbf{R}$ in terms of the available data from the Make and Use tables, we first plug equation (B.5) into equation (B.1) and note that $\hat{g}i_Z = g$ to get

$$q = \mathbf{B}g + e. \tag{B.9}$$

Then, we plug equation (B.6) into equation (B.2) and note that $\hat{q}i_V = q$ to get

$$g - h = \mathbf{D}q. \tag{B.10}$$

From equations (B.7) and (B.10),

$$g = (I_Z - \hat{p})^{-1}\,\mathbf{D}q. \tag{B.11}$$

Plugging in to equation (B.9) gives

$$q = \left(I_V - \mathbf{B}\left(I_Z - \hat{p}\right)^{-1}\mathbf{D}\right)^{-1} e, \tag{B.12}$$

and substituting back into equation (B.11) gives

$$g = \left(I_Z - \hat{p}\right)^{-1}\mathbf{D}\left(I_V - \mathbf{B}\left(I_Z - \hat{p}\right)^{-1}\mathbf{D}\right)^{-1} e. \tag{B.13}$$

Thus, we have arrived at

$$\mathbf{R} = \left(I_Z - \hat{p}\right)^{-1}\mathbf{D}\left(I_V - \mathbf{B}\left(I_Z - \hat{p}\right)^{-1}\mathbf{D}\right)^{-1}, \tag{B.14}$$

which can be computed with our data from the Make and Use tables.

Next, we move to mapping value added in each industry z to its final use, either consumption or investment. To start, we compute a $(Z \times 1)$ vector a, where a_z is total value added in industry z. Using the matrices defined above,

$$a = g - \mathbf{U}^T i_V, \tag{B.15}$$

which is simply gross output minus intermediate expenditure in each industry. Then, we define the $(Z \times Z)$ diagonal matrix $\hat{a}$, where $\hat{a}_{zz} = a_z$ and

$$\hat{v} = \hat{a}\hat{g}^{-1}. \tag{B.16}$$

The $(Z \times Z)$ matrix $\hat{v}$ is constructed such that $\hat{v}_{zz}$ is the ratio of value added to gross production in industry z. Now, we can compute the vectors of consumption value added (a^C) and investment value added (a^X) as

$$a^C = \hat{v}\mathbf{R}e^C, \tag{B.17}$$

$$a^X = \hat{v}\mathbf{R}e^X. \tag{B.18}$$

For intuition, we focus on consumption. Vector e^C gives final consumption expenditure for each commodity, and pre-multiplying by the total requirements matrix $\mathbf{R}$ gives the total production from each industry needed to generate the consumed commodities. Pre-multiplying by $\hat{v}$ gives the fraction of that industry-level output that is value added, rather than expenditure on intermediates. The interpretation is identical for the investment equation. Equations (B.17) and (B.18) assume that input requirements and the ratio of value added to gross output is the same for both consumption and investment, reflecting the principles of proportionality and homogeneity.

Finally, to map our results to the model, we collapse the $(Z \times 1)$ vectors a^C and a^X into the five sectors in our model. The IO codes corresponding to each sector are:

- Agriculture: 111CA, 113FF
- Construction: 23
- Energy and mining: 211, 212, 213
- Manufacturing: 311FT, 313TT, 315AL, 321, 322, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335, 3361MV, 3364OT, 337, 339
- Services: all remaining NAICS codes listed in the make and use tables.

The 2019 sector shares of consumption value added plotted in Figure 1 equal the value added in consumption produced in each of the five sectors divided by total consumption value added. Similarly, the sector shares of investment value added equal the value of investment value added produced in each of the five sectors divided by total investment value added.

B.2 Aggregate Damage Function

In our quantitative application, we compare the effects of climate change in the DICE-like and HD models from 2019, the first period of the simulation, onward. To ensure that the DICE-like and HD specifications generate the same damage in year 2019, we hold the sector-specific damage functions constant at their 2019 values throughout the simulation of the DICE-like model. Specifically, damage in each sector j of the DICE-like model equals $D^{\text{Agg}}(T_t) \times D_j(T_{2019})$. Otherwise the model is unchanged.

To construct $D^{\text{Agg}}(T_t)$, we follow the existing climate-economy literature and take the weighted arithmetic average of sector-level climate impacts, where the weights are given by outcomes in the absence of climate change (e.g., Nordhaus and Boyer, 2003;

Barrage, 2020). In our specification, the aggregate damage function measures climate damage that occurs *after* year 2019. Therefore, we must first measure sector-specific climate damage relative to its 2019 value. Let $\hat{D}_j(T_t) \equiv D_j(T_t)/D_j(T_{2019})$ denote this normalized climate damage in sector j. The aggregate damage is then given by

$$D^{\text{Agg}}(T_t)^{1-\theta} = \sum_j \text{share}_{j,t} \cdot \hat{D}_j(T_t)^{1-\theta} = \sum_j \text{share}_{jt} \cdot \left(\frac{D_j(T_t)}{D_j(T_{2019})} \right)^{1-\theta}, \tag{B.19}$$

where,

$$\text{share}_{jt} = \left. \frac{p_{jt} y_{jt}}{Y_t} \right|_{T_t = T_{2019} \ \forall t}, \tag{B.20}$$

is the equilibrium expenditure share for sector j for a simulation of the model with constant climate. We do the aggregation raising everything to the power of $1-\theta$ to be consistent with the existing literature, which defines damages as factor-neutral productivity terms. By construction, $D^{\text{Agg}}(2019) = 1$ and all three simulations of the model (constant climate, heterogenous damage, and DICE-like) have identical outcomes in 2019, the first period of the simulation.

B.3 Wet Bulb Globe Temperature

The WBGT is an indicator that combines the effects of the ambient air temperature, humidity, sunlight and wind speed to capture the conditions that the human body actually experiences while preforming work outdoors. Directly measuring the WBGT requires complex and expensive instruments, such as the one in Figure B.1, produced by the measurement company, TSI.

The left sensor in Figure B.1 measures the globe temperature. It contains a thermometer inside of a hollow metal sphere that is painted black. As the sphere sits in the sun, it heats up, capturing the the effects of sunlight and radiative heat transfer on the human body. The middle sensor measures the natural wet bulb temperature. It is a thermometer that is fitted with wet wick. The wick is fully exposed to the environment, and thus captures the effects of evaporative cooling on the human body. The right sensor is a thermometer protected by a sun shield, which measures the ambient air temperature. The WBGT combines these three measurements to create

Figure B.1: Wet Bulb Globe Temperature Monitor

Note: The figure shows heat stress monitor produced by the measurement company TSI (downloaded from https://tsi.com/products/heat-stress-monitors/tsi-quest-questemp-32-34-36-area-heat-stress-monitors/).

an indicator of the potential for heat stress.

Standard meteorological data do not report the WBGT. Instead, we use the heat-transfer model developed by Liljegren et al. (2008) to predict the WBGT from data on relative humidity, solar irradiance, wind speed, and the ambient air temperature. The model consists of a complex series of physical relationships. Liljegren et al. (2008) originally programmed the model in FORTRAN. An R-package is also available on Git-hub. We adapted the R code to Matlab to use for our study.

We use state-level data on average relative humidity by month from NOAA. We set wind speed equal to 1 m/s, approximately the amount of wind the body generates by completing physical tasks. To be conservative, we set the solar irradiance to zero and assume instead that workers can adapt to avoid the effects of the sun by performing outdoor work in the shade or at night.