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The U-shape of over-education? Human capital dynamics & occupational mobility over the life cycle

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Abstract

This paper analyzes the relationship between age and the skill requirements of jobs performed by workers. I document that the proportion of college degree holders working in occupations that do not require a college degree is U-shaped over the life cycle and that there is a rise in transitions to non-college jobs among prime age college workers. The downward trend at initial stages of the life cycle is consistent with workhorse models of labor mobility, however, the rising trend at middle stages of the career is not. Such movements down the occupation ladder are also accompanied by average wage losses of 10% from the previous year. I develop an equilibrium model of frictional occupation matching featuring skill accumulation and depreciation along with worker and firm heterogeneity that can match the life cycle profile of downward occupational mobility. The model shows that skill depreciation is the key driver of transitions to low skill jobs with age. Using the model, I simulate the impact of different types of structural change in the labor market and find that the welfare consequences of long term changes depend on the interaction of the life cycle and human capital investment dimension.

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1 Introduction

Unemployment has been a major focus of macroeconomic models of the labor market. The workhorse model in the literature, aptly named the DMP model, features equilibrium involuntary unemployment and has been used for various macroeconomic questions related to the labor market. However, within this literature there has been less focus on "unsuitable" employment in the labor market, such as a college graduates waiting tables and most of the research on this state of the labor market has focused on younger workers. This paper focuses on the incidence of one type of unsuitable employment, namely over-education, documents how it evolves over the course of the life cycle and develops a model that provides an explanation for the stylized facts.

I document that the proportion of college graduates working in jobs that do not require a college degree is U-shaped over the life cycle.[1] Around 30 percent of college graduates are working in non-college jobs at age 30. This percentage decreases until age 40 and then starts rising again. By the age of 65, around 35 percent of college graduates are working in non-college jobs. I call these workers over-educated and refer to their state as over-education or over-educated employment.[2] The downward trend at initial stages of the career is consistent with existing models of labor mobility in which mismatch in worker skills and skill requirements of the job decreases over time as workers overcome search and learning frictions. However, the rise at later stages of the career presents a challenge to such commonly used theories of job ladder and career advancement.

Using longitudinal data I show that among college graduates, prime age workers are more likely to move from college jobs to non-college jobs than younger workers. There is a lot of heterogeneity in these transition probabilities across age which does not show up in the aggregate measure of over-education by age. As is well known, job switching declines with time spent in the labor market as workers accumulate occupation or job specific human capital. However, among those who switch occupations during prime working age years, a higher percentage make transitions to lower skill jobs. Hence, the flow of workers into over-educated jobs increases with age explaining the rise in the overall U-shape of over-education after age 40. Furthermore, I document that workers who make these downward switches in occupations suffer average wage losses of 10% and the college wage premium for the over-educated group is significantly lower than that for the matched workers.

The stylized fact on over-education over the life cycle survives various robustness checks

[1] I focus on college graduates since there is a natural "unsuitable" job for this group-jobs that do not require a college degree

[2] *Matched workers* are college graduates employed in jobs that require a college degree.

such as restricting the sample to male full time workers and using alternative measures of over-education.[3] I also show that workers who transition to over-education come from the lower end of the wage distribution among college educated workers and almost half of them make such transitions without an intervening unemployment spell. Finally, using other measures of job quality such as experience requirements, cognitive skill requirements and median wages, I show that non-college jobs performed by college educated workers are similar to jobs held by non-college workers.

There could be two possible mechanisms that may cause a person to be over-educated in his/her job. The worker may be stuck in a low type job because of labor market imperfections. Such a worker would perform better if he/she were reallocated to a higher type job. This phenomenon is usually referred to as mismatch employment in the literature. The second possible explanation could be that the over-educated worker does not have the required skills to work in a high type job. Such a worker would not be classified as mismatched because the skill level of the worker is consistent with the requirements of the job.

To explain the documented empirical facts, I build a life cycle model of occupational matching with search frictions, on-the-job search and evolving worker productivities. Occupations are vertically differentiated with homogeneous firms within each occupation. The match level production function incorporates positive complementaries between worker ability and firm productivity while allowing for higher skill requirements in high skill occupations. Each worker draws his/her initial skills from a distribution that depends on the acquired education level. The key novel ingredient in my framework is that worker skills can be enhanced by investments in worker skills decided jointly by the worker and firm match. These investments along with a fixed depreciation rate and idiosyncratic shocks determine how worker productivities evolve over the course of the life cycle.

The equilibrium features Positive Associative Matching (or PAM) in which high ability workers match with firms in high productivity occupations. Workers with low skill levels are unable to form matches in high skill occupations due to higher skill requirements in those jobs. Employed workers receive offers from firms in other occupations and move to firms with higher match surpluses. On the job training for younger workers makes them more productive and they climb the occupation ladder through on-the-job search or following an unemployment spell. However, as workers become older and approach an exogenous retirement age, the incentive to invest in worker skills decreases. As a result, skill depreciation leads to a net decline in the productivity of the workers and they choose to move to lower rungs of the ladder if an outside offer comes their way. Workers also move down the

[3] These alternative measures are discussed in section 2.4

occupation ladder after exogenous destruction shocks end high productivity matches.

The model is then calibrated to match (i) the over-education profile documented in the data, (ii) wage growth over the life cycle and (iii) the proportion of workers of each education level working in different occupation groups and (iv) the probability to transition to low skill jobs as a function of worker wages. The calibrated model shows that skill depreciation is the key ingredient for explaining transitions to lower skill jobs among prime age and older workers. A model without the fixed depreciation of skills leads to workers moving to higher skill jobs as they become older, a prediction inconsistent with the data.

The model is well suited for studying the effects of structural change in the labor market on the careers of workers and how long term changes interact with life cycle patterns. In a counter-factual exercise, I simulate a particular type of structural change in which the productivity of middle skill occupations is decreased permanently. I find that in comparison to the baseline results, the new steady state features "job polarization" with employment growth in low and high skill occupations at the expense of middle skill occupations. I also find that workers earn higher wages on average in the new economy. This is because high skill occupations gain more employment than low skill ones under this scenario. In another counter-factual exercise I increase the skill requirements of high skilled occupations from the baseline calibration. This also leads to a decline in employment in middle skill occupations but a larger part of the workforce is reallocated to low skill jobs. I find that workers earn lower wages on average in this new economy. This result is driven by the lack of progression up the occupation ladder due to the higher skill requirements of jobs.

1.1 Literature Review

The empirical results in this paper conform with some recent work on the occupational transitions of prime age and older workers. Focusing on occupational mobility within firms, Forsythe (2016) finds that a substantial amount of re-allocation within firms is to lower quality jobs, where the quality of a job is defined by measures taken from the O*NET database and includes the education requirements of jobs. She also documents that for young workers, the predominant move is to high skilled jobs while for older workers occupation changes are mostly towards low skill jobs.

Theoretically this paper is related to models of occupation choice (Jovanovic (1979b)). The main insight from this literature is that workers find their comparative advantage as they try different occupations. Occupations are assumed to be identical in skill requirements but workers have occupation specific ability which they discover over time. As workers sample more occupations they find the match with the highest ability. This mechanism generates worker turnover across occupations. Several papers have used such models to explain empirical regularities about labor turnover such as declining occupation switching by age, increasing wages by tenure and high unemployment rates for the young (Menzio, Telyukova, and Visschers (2012); Gervais et al. (2014)). A recent paper by Groes, Kircher, and Manovskii (2014) emphasizes the role of adding absolute advantage to the theory of comparative advantage. They introduce vertically differentiated occupations in an equilibrium environment to explain occupational mobility patterns across the wage distribution.

The mechanisms present in these models however, cannot generate the empirical patterns documented here. These models will predict that workers move to better matches over time and stay there. This will thus produce a downward sloping profile for over-education over the life cycle instead of a U-shape. To generate the life cycle patterns documented here, I borrow insights from the literature on life cycle wage growth and human capital (see Rubinstein and Weiss (2006), Sanders and Taber (2012) and Huggett, Ventura, and Yaron (2011)). In these models, workers make active human capital investments over their career where the opportunity cost of investment is forgone earnings. Human capital investments decline with age and worker productivity is thus hump-shaped over the life cycle.

On a theoretical level, I combine vertical sorting into occupations with human capital investment and search frictions while endogenizing the vacancy posting decisions of the firms. Most matching models assume that the distribution of attributes on both sides of the market is exogenous and fixed. Recently, some dynamic matching papers have started to relax this assumption and analyze environments where the attributes change based upon the match (Anderson and Smith, 2010). In my setup, the attributes of the occupations stay fixed but the productivity of the workers evolves based on human capital investments. These investments in turn depend on the occupation that the worker is currently matched with and upon his chances of moving up the occupation ladder.

The two most closely related papers to my work are Flinn, Gemici, and Laufer (2016) and Lentz and Roys (2015) which also incorporate investment in worker training by firms in a frictional environment. While the former abstracts from firm heterogeneity, the latter restricts training to be of two types, high and low. In contrast, this paper features ex-ante firm heterogeneity, continuous time investments in worker training from the interval $[0, 1]$ and skill depreciation during unemployment. Finally, compared to the two papers above, I allow unemployed workers to direct their search towards occupations with different production technologies and job finding rates. Moving beyond the technical differences, the motivation of my paper is to explain the occupational choice of workers over the life cycle with an added emphasis on downward mobility while these papers focus on explaining life cycle wage growth and the role of frictions in determining the returns to training.

2 Stylized Facts

2.1 Measuring Required Level of Education for Occupations

I use the Department of Labor's O*NET data to measure education requirements for each occupation. The O*NET program collects data on entry requirements, work styles and task content within occupations by surveying each occupation's working population. For educational requirements, I use the question that asks incumbent workers, "If someone was being hired to perform this job, indicate the level of education that would be required". The survey respondents are reminded that the question is not asking about the level of education that the incumbent has achieved. Respondents are given options such as less than high school, high school, some college, associate's degree, bachelor's degree, etc. To assign a required level of education to each occupation, I use the distribution of responses. If more than 50 percent of respondents within an occupation agree on the required education level then I assign that education category as the requirement. If less than 50 percent of respondents agree on the required level of education then I assign the mode of the responses as the required level of education but only if the difference between the mode and second largest category is greater than 5 percent. If the difference is less than 5 percent then I assume that both education categories can be the required level of education for that particular occupation.[4]

2.2 Measuring Over/Under Education

I combine the education requirement data with survey data on worker characteristics such as the Current Population Survey (CPS). The CPS data contains information on each worker's acquired level of education and the worker's current or most recent occupation. It is also a longitudinal dataset and workers can be observed one year apart and I use this feature to document transition patterns across labor market states by age.

I define two measures of over-education and focus only on individuals with a bachelor's degree or higher. In the first measure, I restrict attention to bachelor degree holders and define them as over-educated if they are working in non-college jobs. College jobs are defined as occupations that require at least a college degree or higher. For my second measure, I use individuals with more than a college education and define them as over-educated if they are working in a non-college job. The latter measure understates over-education at the top of the education distribution because it is highly unlikely that a person with a doctoral degree

[4]In the appendix I use an alternative measure of required level of education in terms of years of education. The results over the life cycle are similar with that measure as well.

is working in a non-college job. Nevertheless, I use this measure to avoid misclassification of highly educated workers as over-educated. Most of the results on the second measure are reported in the appendix.

The method used for measuring education requirements of occupations is consistent with the approaches taken in the over-education literature (Leuven and Oosterbeek, 2011). It also matches up well with a subjective measure of over-education from the National Survey of College Graduates. The O*NET database matched with CPS has also been used by Abel and Deitz (2015) to determine the aggregate level of over-education in the U.S economy and how it evolves over the business cycle. In the appendix, I show that the over-education measure used by Abel and Deitz (2015), which uses a different definition of college jobs, produces similar patterns over the life cycle.

2.3 Over-Education over the Life cycle

I now present my main empirical finding regarding over-education over the life cycle. I use cross-sectional data from the Current Population Survey-Merged Outgoing Rotation Groups (CPS-MORG) to report the proportion of people of each age group who are over-educated in the years 2003-2010. The choice of time period is based upon the timing of the collection of education requirements in the O*NET data which started during the 2000s. Robustness results from the PSID are presented in the appendix in which I follow workers longitudinally for multiple years to produce life cycle profiles of over-education and find similar results to cross-sectional data.

2.3.1 Evidence from Current Population Survey

My benchmark method to estimate the life cycle profile of over-education is to perform a kernel-weighted local polynomial regression of the over-education status on the age of the individual. I choose a bandwidth of 5 and thus the results are similar to regressing the over-education status on dummy variables for 5 year age bins (without a constant) and plotting a best fit line through the co-efficients. I restrict the analysis to workers who are currently employed. All regressions are weighted by the cross-sectional weights and the number of hours worked by the respondent.

I find that, for bachelor degree holders, the incidence of over-education by age is U-shaped, as can be seen in Figure 1. Before age 30, more than 30 percent of bachelor degree holders are over-educated in their jobs. This proportion drops below 30 percent by age 40 as workers end up getting matched with jobs that require their level of education. However, the over-education ratio starts rising after age 40, modestly at first and rapidly after age 50. The rise is such that by age 65 (the usual retirement age), there are more over-educated bachelor degree holders than there are at age 30. This fact is quite striking, especially with all the focus on the young college graduates not being able to secure good jobs. It seems that a higher proportion of workers suffer the same fate at later stages of their careers.

2.4 Robustness Checks

2.4.1 The U-shape of Over-Education for a Restricted Sample

Readers can perhaps question whether the pattern above is driven by particular demographic groups. In this section I repeat the analysis by restricting the sample to male full time workers. The results are shown in Figure 2. As can be seen the patterns across age for this sample are also very similar to the overall sample.

2.4.2 The U-shape of Over-Education after Controlling for Demographics and Year Fixed Effects

While it is true that being a female or a part-time worker has a positive impact on the incidence of over-education, the age profile of over-education after controlling for demographic characteristics is still U-shaped. In this section, I control for other demographic characteristics that might be important in explaining the incidence of over-education along with age. I also control for year fixed effects to show that this phenomenon is not driven by aggregate booms and busts. I then report the marginal effects with respect to age which can be interpreted as the residual effect of age on the incidence of over-education after controlling for demographics and year fixed effects. More specifically, I divide individuals into 5 year age bins and then estimate the following regression:

$$Y_{ia} = \beta_0 + \sum_{a=1}^{a=9} \beta_a D_{ia} + \gamma X_i + \delta_t \epsilon_{ia},$$

where Y_i is an indicator of over-education which equals 1 if person i is over-educated, and D_{ia} is a dummy variable which is 1 if individual i belongs to age group a. Demographic control variables are in the vector X_i which contains dummy variables for gender, marital status, self-employment status and a dummy variable for whether the individual was born in a foreign country. I plot the marginal effect of age on the incidence of over-education in Figure 3. As can be seen, the probability of being over-educated first declines and then rises with age. The results are thus similar to the ones presented in the previous sections where the proportion of over-education was U-shaped.

2.4.3 Evidence from National Survey of College Graduates

To provide corroborating evidence I use the National Survey of College Graduates (NSCG). The NSCG is conducted by the National Science Foundation and only contains college graduates, i.e., individuals with at least a bachelor's degree. Respondents who are employed at the time of the survey are asked the following question:

"Did your duties on this job(current job) require the technical expertise of a bachelor's degree or higher in ... ".

- ↪ Engineering, computer science, math or the natural sciences
- ↪ The social sciences
- ↪ Some other field (e.g., health business or education)-Specify

Respondents are asked to mark Yes or No for "each" item. I classify respondents as over-educated if they answer No to all three items. Notice that this measure is similar to the one developed above, where I defined some occupations as non-college jobs and defined over-education as college graduates working in non-college jobs. Thus, the life cycle profile of over-education from this measure should be the same as documented before using an objective measure. I use the NSCG samples from the years 2003, 2008 and 2010 in my empirical analysis.

Figure 4 provides evidence on over-education among college graduates in the NSCG. The magnitudes and the U-shape is similar to Figures 1 and 2 which shows that the patterns in Figures 1 and 2 are not driven by the method used to construct education requirements using the O*NET data.

2.4.4 Transitions across Labor Market States

In this section I use the panel dimension of CPS-MORG data to document the transitions to and from the over-education state over the life cycle at yearly intervals. The results are shown in Figure 5 which shows that the probability of moving towards over-education increases with age. [5] Panel (a) shows that workers with a bachelor degree are more likely to move towards over-education with age. While transitions in the other direction decrease after the age of 40 as seen in panel (b). A noticeable feature of Figure 5 is that there is a lot of

[5] The over-educated state is referenced by "OE" while the matched state is abbreviated by "M".

heterogeneity in the transition probabilities by age which is masked in Figures 1 and 2. The probability to transition to over-education increases by 50% over the course of the life cycle while the probability to transition to matched state from over-education increases by 25% over the life cycle. Such large changes cancel each other out in the aggregate which leads to a change of 3% over the life cycle in the probability of being over-educated. Figure 6 shows these transitions conditional on an occupation switch. As can be seen the probability of moving from over-education state to the matched state conditional on changing occupations is declining throughout the life cycle. On the other hand transitions to the over-education state stays relatively flat with age. Taken together these figures suggest that the U-shape of over-education with age observed in the cross-sectional data is driven by an increased flow of workers into over-education and a decreased outflow in the other direction.

2.5 Implications of Over-education for Wages and Experience

Having established that college graduates make transitions to low skill jobs during prime working age, this section documents two additional facts associated with over-education. The literature has already documented that at the individual level, over-education is associated with lower wages and I corroborate this evidence across the life cycle in appendix Figure A5. Here, I go one step further and show that workers who make transitions to over-education suffer real wage losses of around 10%. This would allay fears that transitions to over-education that I have presented before do not represent a movement down the occupation ladder. I also document that over-educated workers are more likely to be working in "Entry Level" jobs throughout the life cycle and there is no evidence that older over-educated workers are working in jobs that require more experience.

2.5.1 Wage effects

One advantage of using the CPS-MORG data is that they have information on a worker's weekly earnings and usual hours of work. Using this information one can construct the hourly wage rate for all employed individuals in the sample. Since I have data from multiple years, I construct real wages in 1999 dollars and then estimate wage growth from one year to the next for workers making different types of transitions. To document how wage losses upon transition to over-education differ with age, I also interact the transition to over-education dummy with the age variable. More specifically I estimate the following equation:

$$\Delta logw_i = \beta_0 + \beta_1 \mathbf{1}\{\text{M} \rightarrow \text{OE} = 1\} + \beta_2 \mathbf{1}\{\text{OE} \rightarrow \text{M} = 1\} + \beta_3 \mathbf{1}\{\text{Occ change}\} + \sum_{a=1}^{a=9} \gamma_a D_{ia} + \sum_{a=1}^{a=9} \delta_a D_{ia} \times \mathbf{1}\{\text{M} \rightarrow \text{OE} = 1\} + \lambda_t + \theta X_i + \epsilon_i$$

where β_1, β_2 and β_3 measure the effect of making a transition to over-education, making a transition to a matched job and making a occupation switch respectively. Furthermore, I add age dummies, year fixed effects, other demographic controls and interact the age dummies with the dummy variable for making a transition to over-education. The equation was estimated jointly for all college graduates [6] and I show the marginal effect of age on wage growth for workers making a transition to over-education and those who do not in Figure 7. While wage growth typically declines after age 40, those making a transition to over-education suffer wage losses of around 10 % even at the age of 45. For comparison, the wage growth for workers not experiencing a transition to over-education at 45 is about 1 %.

2.5.2 Experience Requirements for Over-educated Workers

It might be the case that over-educated old age workers are working in jobs that require high experience and thus the jobs are different than the ones held by over-educated young workers. This would imply a tradeoff between skill and experience in the labor market. To test this hypothesis, I use the O*NET data to determine the experience requirements for each occupation. The O*NET data program asks the following question from incumbents about their occupations:

"If someone was being hired to perform this job, how much RELATED WORK EXPERIENCE would be required? (that is having other jobs that prepare the worker for the job)".

The answer is based on a 12 point scale with the values less than 5 indicating less than one year of required experience (potentially entry level jobs) and values greater than 10 indicating at least ten years of related work experience in similar jobs. Thus, using the methods described above for calculating education requirements for each occupation, I also determine the experience requirements for each occupation and merge it with CPS data. I then calculate the experience requirements of jobs held by over-educated and matched workers at different stages of their careers. The results of this exercise can be seen in Figure 8. Older workers who are over-educated are working in jobs which are similar to the jobs done

[6] The equation is not estimated separately for bachelor degree holders and those with higher degrees.

by young over-educated workers in terms of experience requirements and they are mostly entry level jobs. Thus, older over-educated are also over-experienced in their jobs.

Showing that over-educated workers suffer wage losses upon making a transition and that they are not working in jobs that require a lot of experience shows that I have identified non-college jobs correctly using my measure of over-education. I provide further evidence in the appendix that this measure does a very good job of capturing differences across college and non-college jobs in various dimensions such as median wages, cognitive skill requirements of occupations and earned wages.

2.6 Who Transitions to Over-education and How?

To wrap up my stylized facts, I show that the probability of transitioning to over-education is monotonically decreasing in past wages and cognitive skill requirements and that such transitions do not occur only after unemployment spells. Figure 9 shows the probability of transitioning to over-education for male full time workers over the age of 35 as a function of their relative wage among workers working in college jobs. As can be seen workers at the lower end of the wage distribution are more likely to transition to over-education. This fact informs the model that I consider in the next section. Appendix Figure A3 shows the rank of these workers (who made a transition to over-education) in the wage distribution of non-college jobs. Workers who made a transition to over-education are more likely to end up earning a higher wage relative to all workers in non-college jobs.

I also measure the probability of transitioning to over-education as a function of the skill requirements of the past occupation. The measure of skill requirements in a job is taken from Acemoglu and Autor (2011) and it measures the cognitive skills required to perform a job. Acemoglu and Autor (2011) argued that college educated workers are more likely to work in occupations that require more cognitive skills. I divide college occupations into 10 bins based on this measure. Thus, occupations in the 10th bin require the most cognitive skills among jobs that require a college degree.

I, then, estimate the probability of transitioning to over-education as a function of the skill requirement in the past job for male workers aged 35 and above who work more than 40 hours. The results are shown in Figure 10. Similar to Figure 9, workers in the lower end of the skill distribution are more than twice as likely to transition to over-education. Figure A4 in the appendix shows that their most likely destination is the higher end of the skill distribution amongst non-college jobs.

Finally, I show that such downward transitions do not necessarily come after unemployment spells. In fact, as shown in Figure 11, almost half of the college workers moving to over-education had a job in the previous month. This fraction rises to 70% if one does not consider out of labor force workers as unemployed. The fact that a significant portion of these downward transitions happen without an intervening unemployment spell calls for a model which allows for job-to-job transitions. There is also significant upward mobility in terms of education requirements in the data, as can be seen from Figures 5 and 6, and most of such movements take place without intervening unemployment spells. This is another reason why job-to-job transitions should be allowed in a model which aims to explain movements up and down the job ladder in terms of education requirements.

3 Model

In this section, I present an equilibrium model of life cycle occupation search, with heterogeneous workers and firms, skill accumulation, idiosyncratic uncertainty and on-the-job search. Workers and firms encounter frictions in the matching process as in the canonical DMP model and they jointly decide how much to invest in worker skills.

3.1 Framework

Time is discrete and continues forever. There is a finite number of occupation submarkets indexed by $k = 1, 2, ...K$ which differ in terms of their production function, job finding probabilities and skill accumulation technology. Occupations are ranked in terms of their productivity with p_k being the productivity of the k^{th} occupation and $p_1 < p_2 < p_3 .... < p_{K-1} < p_K$. Firms within each occupation submarket are assumed to be homogeneous and have access to the same production and skill accumulation technologies.

Each worker stays in the labor market for T periods and the age (or the time spent in the labor market) of the worker is indexed by t. Workers possess general human capital, h, which can be transferred across occupations and can be referred to as the skill or the productivity of the worker. The type of the worker can be summarized in the double $x = (h, t)$.

Workers are assumed to be risk neutral and discount the future at rate β. They choose to search in different occupations over time to maximize the sum of their discounted lifetime earnings. Unemployed workers have access to unemployment benefits which depend on the skill of the worker. Each occupation submarket has a DMP structure in which workers and firms match, production takes place, surplus is split and continuation decisions are made.

All workers enter the labor market with a starting level of productivity which is correlated with their level of education. When employed, a worker's productivity evolves endogenously based on the investment decisions made by the worker and firm within a match. Following the literature on endogenous human capital accumulation, it is assumed that each worker possesses a unit amount of time each period. This can be allocated to investments in human capital s, which lead to higher productivity in the future or to production activities $(1-s)$. In particular, a human capital evolution function is specified, $h' = g(s, h, z)$, which maps current human capital h to future human capital h' based on the investment decision s and shocks to skill accumulation, z. The level of worker skills that can be used in the production process is then $e = (1-s)h$. Thus, the workers accumulate human capital by learning on the job as opposed to learning by doing. The key innovation of the current setup is that the investment decisions are not made by the worker but jointly by the worker and the firm as an outcome of a generalized Nash Bargain. [7] Workers do not accumulate human capital when unemployed.

Once matched within an occupation, the worker and the firm produce according to a occupation specific production technology. Defined at the match level, the production function combines worker skills and the productivity of the firm to create value added $f(e, p_k) \subset R$. I allow for the possibility that positive value added may require a threshold level of input from the worker. Thus, firms operating in higher productivity occupations might require workers to provide a minimum level of skill before they make positive profits. Another way to state this assumption is that high productivity jobs can only be performed by workers above a certain skill level. Such a restriction on the production technology has been used in the literature previously by Albrecht and Vroman (2002) and more recently by Lise and Robin (2014). Furthermore, I allow for complementaries between the worker and firm types, $f_{e,p_k} \geq 0$.

3.2 Hiring, Poaching and Separations

Unemployed workers can direct their search to different occupation submarkets while employed workers get random offers while employed. Unemployed workers searching in occupation k contact a vacancy with probability λ_k. Employed workers working in occupation k get a job offer with probability λ_0 and the job offer is from occupation $l \neq k$ with probability

$$\eta_l = \frac{\lambda_l}{\sum_{i \neq k} \lambda_i}$$

Once a worker employed at a firm in occupation k receives an offer from a firm in occupation l, the worker ends up at the firm with the higher match surplus.

[7] Such a setup has previously been formulated in Sanders and Taber (2012).

Wages and investment decisions are contingent upon the worker's type and also (potentially) on the outside option of the worker. For unemployed, the outside option is the value of unemployment and for employed, the outside option is either the total match value offered by a dominated firm (the one with a lower match surplus) or the value of unemployment if the worker has no offer from another firm. Hence, the value functions of the worker and the firm depend upon the type of the worker, the type of the firm and the outside option of the worker. Denote by $W_k(x, i)$ as the value function of a worker of type x working in occupation k with outside option i. Similarly, denote by $J_k(x, i)$ as the value function of a firm in occupation k in a match with a worker of type x with outside option i. Value of unemployment is denoted by $U(x)$ and the value of an open vacancy by V_k.

Define the surplus of a match between a worker of type x and a firm of type k as the sum of the surplus to the worker plus the surplus to the firm:

$$S_k(x) = W_k(x, i) - U(x) + J_k(x, i) - V_k \tag{1}$$

Here I am already assuming that the surplus is independent of the outside option of the worker. This is a standard result under the assumption of transferable utility and it will be shown later that this is indeed the case. Each period matches may end due to exogenous and endogenous reasons. Endogenous separation decisions are jointly efficient which implies that a worker and firm match ends if the match surplus is negative. Matches can also end due to exogenous reasons with probability δ. The separation probability can thus be described by the following function:

$$d_k(x) = \begin{cases} \delta, & \text{if } S_k(x) > 0 \\ 1, & \text{otherwise} \end{cases} \tag{2}$$

Based on the description of on-the-job search above, separation to another occupation is described by the following decision rule:

$$f_{l,k}(x) = \begin{cases} 1, & \text{if } S_l(x) > S_k(x) \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

3.3 Worker's Problem

Now consider an unemployed worker characterized by the pair $x = (h, t)$ at the start of the period. The value function of the worker is given by:

$$U(x) = \max_{k(x)} bh + \beta \mathbb{E}_{x'|x,u} \left\{ (\lambda_k W_k(x', u) + (1 - \lambda_k) U(x')) \right\} \tag{4}$$

$$
\begin{aligned}
h' &= g_u(h, z) \\
t' &= t + 1 \\
U(x) &= bh \quad \text{when} \quad t = T
\end{aligned}
\tag{5}
$$

where λ_k denotes the job finding probability in occupation k and $g_u(h, z)$ is the human capital evolution function during unemployment which depends upon the current productivity of the worker h and an exogenous shock process z. In the last period of the life cycle when $t = T$, the worker receives the flow value of unemployment and no continuation value.

The value of unemployment consists of the flow value of unemployment benefits, which are a linear function of worker human capital, and the discounted expected value at the start of next period. In the next period with probability $(1 - \lambda_k)$, the worker stays unemployed and with probability λ_k he finds a job in occupation k. In the latter scenario, the value function of the worker is denoted by $W_k(x, u)$ where the state variable u indicates that the outside option of the worker during bargaining was his value of unemployment. Workers choose the occupation k that maximizes their value today given their state variables. There is no direct (explicit flow cost) or indirect (through loss of human capital) reallocation cost to workers for switching occupations and thus they can switch to a new occupation in the next period. The occupation choice function associated with the above problem is $k(x)$.

Now consider an employed worker with state $x = (h, t)$ employed in occupation k. The value of employment depends upon the attributes of the worker, the type of the firm and the firm he or she uses as the outside option in Nash Bargaining. Using the terminology of Jarosch (2014), I refer to the latter firm as the "negotiation benchmark". I assume that when the worker receives no job offer when employed, wages and investment decisions are renegotiated and the negotiation benchmark becomes unemployment as is the case when the worker is hired out of unemployment.

The worker and the firm jointly agree upon the level of investment $s_k(x)$ which impacts worker productivity in the next period through the human capital production function. The units of worker skill used in the production process are given by $e_k(x) = (1 - s_k(x))h$. The expected value of employment for a worker of type x, matched to a firm of type k with negotiation benchmark i and investment in training, $s_k(x)$, is given by:

$$
\begin{aligned}
W_k(x, i) = &w_k(x, i) + \beta \mathbb{E}_{x'|x,k} \Big\{ d_k(x')U(x') + (1 - d_k(x')) \\
&\big\{ \lambda_0 \sum_{l \neq k} \eta_l (f_{l,k}(x') W_l(x', k) + (1 - f_{l,k})(x') W_k(x', l)) \\
&+ (1 - \lambda_0) W_k(x', u) \big\} \Big\}
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
h' &= g_k(h, s_k, z) \\
t' &= t + 1 \\
W_k(x, i) &= w_k(x, i) \quad \text{when} \quad t = T
\end{aligned}
\tag{7}
$$

The worker receives an outside offers in another occupation at rate λ_0. If the outside offer is from occupation l and $S(x, l) > S(x, k)$ then the worker moves to the firm of type l and firm k becomes the negotiation benchmark. On the other hand if $S(x, l) < S(x, k)$ then the worker stays with his current firm but firm l becomes the negotiation benchmark. At T, the worker receives the current wage and exits the labor market at the end of the period.

3.4 Firm's Problem

Consider the expected profit of firm in occupation k employing a worker of type $x = (h, t)$ and negotiation benchmark i assuming investment policy $s_k(x)$:

$$
\begin{aligned}
J_k(x, i) =& f(e_k(x), p_k) - w_k(x, i) + \beta \mathbb{E}_{x'|x,k} \Bigg\{ (1 - d(x')) \\
& \Big\{ \lambda_0 \sum_{l \neq k} \eta_l ((1 - f_{l,k}(x')) J_k(x', l)) + (1 - \lambda_0) J_k(x', u) \Big\} \Bigg\}
\end{aligned}
$$

where $d_k(x')$ is the separation decision defined above and is equal to 1 if the match surplus is negative. Otherwise matches break up with the exogenous probability δ. If the worker receives an outside offer from firm of type $l \neq k$ and $S(x, l) > S(x, k)$, the worker moves to firm l and firm $k's$ continuation value is given by V_k which is assumed to be equal to 0 in equilibrium and hence not presented in the firm value function above.

The amount of output produced by a worker firm pair depends on the production technology available to the firm in occupation k and the amount of worker skill used in the production process.

3.5 Bargaining, Wages and Investment Decisions

I assume that wages and investment decisions are determined by generalized Nash Bargaining. Following Dey and Flinn (2005) and Cahuc, Postel-Vinay, and Robin (2006) I assume that when a worker encounters an outside offer, the worker moves to the firm with the higher match surplus and his outside option is the total match value offered by the dominated firm. This is the maximum value that the dominated firm can offer to the worker. When the worker does not have an outside offer or is hired from the unemployed pool, his outside option is the value of unemployment.

Define $M_k(x)$ as the total value of the match between worker of type x and firm of type k. This is equal to the sum of the value to the worker plus the value to the firm. Now consider a worker firm match in occupation k with worker type x and worker outside option $M_i(x)$ (total surplus from dominated firm i or the value of unemployment) that produces a positive surplus. The wage, $w_k(x,i)$, and investment, $s_k(x)$ solve the generalized Nash bargaining problem:

$$(w_k(x,i), s_k(x)) \in \arg\max \left[W_k(x,i) - M_i(x)\right]^q \left[J_k(x,i) - V_k\right]^{1-q} \tag{8}$$

where $q \in [0,1]$ is the exogenously specified bargaining power of the worker. Lemma 1 establishes a useful result.

Lemma 1. *$s_k(x) \in \arg\max S_k(x)$ iff $s_k(x)$ solves (8)*

Proof. Imposing the equilibrium free entry condition which leads to $V_k = 0$, the wage function $w_k(x)$ solves:

$$W_{k,i}(x) - M_i(x) = q[J_{k,i}(x) + W_{k,i}(x) - M_i(x)] = q[S_k(x) - S_i(x)] \tag{9}$$

Similarly, one can show that the wage function also solves the following equation

$$J_{k,i}(x) = (1-q)[S_k(x) - S_i(x)] \tag{10}$$

Substituting equations (9) and (10) into (8), the problem reduces to:

$$\begin{aligned} & s_k(x) \in \arg\max q^q(1-q)^{(1-q)}[S_k(x) - S_i(x)] \\ \iff \; & s_k(x) \in \arg\max S_k(x) \end{aligned} \tag{11}$$

□

Due to the bargaining protocol the current firm k takes the surplus of the match with firm i as given and hence the best response of firm k is to choose the level of investment to maximize its own surplus. Thus to determine the investment for each worker firm pair and the mobility decisions of the workers, it is useful to work with the surplus function rather than the individual value functions of the firm and the worker.

The surplus function can be written explicitly as

$$S_k(x) = \max\Bigg\{0, f(e(x), p_k) - bh + \beta\mathbb{E}_{x'|x,k}[(1-d(x'))\{\eta_i \mathbf{1}_{\{S_i(x')>S_k(x')\}} q(S_i(x') - S_k(x')) + S_k(x')\} + U(x')] - \beta\mathbb{E}_{x'|x,u}[U(x') + q\max_{j(x')} \lambda_j S_j(x')]\Bigg\} \tag{12}$$

where the expectation operator is dependent on the state of the worker as human capital evolves differently during employment and unemployment. Note that the surplus function depends only on the attributes of the current firm and the worker and not on the type of the firm used as the negotiation benchmark.

The equation for the surplus function can be solved jointly with the value function for unemployment which can be rewritten as:

$$U(x) = bh + \mathbb{E}_{x'|x,u}\left[U(x') + q\max_{k(x')} \lambda_k S_k(x')\right] \tag{13}$$

3.6 Equilibrium

For the quantitative exercise in the next section, I consider the long run stationary equilibrium of the model economy and match data moments to model moments from the stationary equilibrium to calibrate model parameters. In a stationary equilibrium, the decisions of the workers are only dependent upon their type and not upon the distribution of workers in various states of the labor market. Similarly, the decisions of the firms depend upon the occupation in which they operate and the type of the worker they are matched with.

A stationary equilibrium is a set of value functions $U(x), W_k(x,i), S_k(x)$, occupation choice function $k(x)$, separation decision $d(x)$, wage function $w_k(x)$, investment functions $s_k(x)$ and laws of motion for the distribution of employed and unemployed workers over all states of the model such that:

1. The value functions satisfy equations (4), (6), (13) and (14).

2. Wages and investment decisions solve the generalized Nash bargaining problem (9).

3. The distribution of unemployed and employed workers across occupations is stationary and consistent with the policy functions above, shocks to the stock of human capital and job destruction shocks.

4 Quantitative Exercise

For the quantitative exercise I assume that there are three occupation sub-markets with $P_3 > P_2 > P_1$ and label 2 and 3 as college occupations while occupation 1 is referred to as non-college occupations. Within college occupations, occupation 3 refers to occupations that require more than a bachelors' degree. On the worker side heterogeneity comes from variation in initial human capital, h_0. I assume that workers with different education levels draw their initial productivity from the same distribution but with different means. These education levels or worker types are restricted to no-college workers (denoted by nc), bachelor degree holders (denoted by b) and workers with more than a college education (denoted by mc). These three types of workers draw initial human capital from log-normal distributions with mean μ_i, such that $\mu_{mc} > \mu_b > \mu_{nc}$, and variance 1. Hence, the quantitative exercise maps the observable level of education to unobservable worker productivity h and the model traces out the life cycle path of h which determines the occupations that workers work in.

4.1 Parametrization

The model period is set to one quarter and the workers are assumed to stay in the labor market for 160 time periods which implies a working life of 40 years. Value added at the match level in each occupation is parameterized in the following way:

$$f(e, p_k) = \tau_{1,k} e p_k - \tau_{0,k}$$

where I restrict $\tau_{1,k} = 1$ so that $f_{e,p_k} \geq 0$ and $\tau_{0,k} \geq 0$ and its value is to be estimated. This allows for the possibility that firms with higher p_k may operate with more costly non-labor inputs. If that is the case then only workers above a certain level of productivity would be able to deliver positive value added to the firms even if all their time is devoted to the production process and not divided between production and investment in human capital.

For the human capital transition function, I specify a functional form consistent with the literature that seeks to explain wage growth over the life cycle. In particular, the human capital transition function in occupation k is given by:

$$h' = g_k(s, h, \sigma, z) = exp(z) A_k (sh)^{\alpha} + (1 - \sigma_k) h$$

In the above specification, σ_k refers to the depreciation rate of human capital and A_k is referred to as the learning ability. I allow for both the learning ability and depreciation rate to be occupation specific.[8] I assume that worker skills cannot be augmented while the worker is unemployed. Idiosyncratic shocks to worker skills are captured through z which are i.i.d draws from a random normal distribution whose parameters have to be calibrated.

A direct consequence of this parametrization is that if $\tau_{0,k}$ is larger for high productivity occupations then young workers with lower human capital search and work in low productivity occupations, increase their productivity through costly investments and then move up the occupation ladder to higher productivity occupations. Similarly as workers get older, investments in human capital decline and depreciation leads to a fall in overall worker productivity which leads to workers separating from their matches in high productivity occupations and movement towards occupations with lower skill requirements.

4.2 Calibration

Some parameters of the model are set exogenously. In particular, the job finding probabilities for each occupation, λ_k, are calculated from the CPS data using the flows based approach of Shimer (2012). However, calculating the job finding probabilities for each occupation consistent with the definition in the model is not possible using CPS data. That is because when a worker is classified as unemployed in the CPS data, he is assigned the occupation that he was last working in. This may or may not be the occupation that he is currently searching in and this can lead to mis-measurement of the job finding rate for each occupation.

To circumvent this issue, I calculate the job finding probability for each occupation by education groups. The crucial assumption is that most non-college workers search in non-college occupations and college educated workers search in college occupations. Using this approach I find that $\lambda_1 > \lambda_2 > \lambda_3$. Moreover, it is always the case that non-college jobs are more easier to find than all college jobs. An alternative approach could be to estimate the job finding rates of each occupation group with the rest of the parameters by targeting transition rates into each occupation from employment and unemployment.

The rest of the parameters of the model are estimated to match certain moments from the data. The chosen moments include the fraction of people with bachelor degrees working in non-college occupations (or OE workers) by 5 year age bins, proportion of more-than-college workers who are in college jobs (or matched *mc* workers), proportion of non-college workers working in college jobs (or under-qualified *nc* workers), proportion of bachelor degree holders working in occupations requiring more than a bachelor's degree (occupation group 3), proportion of more-than-college workers working in occupation group 3, ratio of wages of

[8]This is a departure from the literature on life cycle wage growth which assumes that ability is correlated with initial ability of the worker and depreciation rate is constant across individuals

OE workers to non-college workers, an unemployment rate of 5%,probability of transitioning to over-education as a function of worker's past wage percentile and life cycle wage growth. The probability of transitioning to over-education is normalized to 1 in the lowest percentile. Similarly, to capture wage growth over the life cycle wages are normalized to 1 for the youngest age group.

I now provide an informal identification argument that defends the moments chosen from the data. The proportion of workers in each occupation along with the job finding rates, helps identify the parameters of the production function and those of the initial distribution. The overall U-shape of over-education is informative about the human capital accumulation and depreciation process and provides information to identify both the production and the human capital evolution function parameters. These parameters are also disciplined by wage growth over the life cycle. The values for $\tau_{0,k}$ and σ_k are also directly related to the relationship between past wages and the probability to transition to over-education. The unemployment rate in the model depends upon the generosity of unemployment benefits conditional on the job finding rates and the parameters of the production function. Thus, the value of the unemployment rate helps identify the value of the parameter b.

Table 1 and Figure 12 show the fit of the model along these moments. The model does a good job of matching the overall shape of the life cycle profile of over-education observed in the data (see table 1). However, it does over-predict the fraction of over-educated workers in the youngest age group. The model also matches the life cycle wage growth profile as well as the share of workers from different education groups working in occupations requiring college education or more. It also captures the empirical fact that the wages earned by over-educated workers are close to the wages earned by non-college workers, the ratio in the model being 1.05. The model also captures the decline in probability to transition to over-education as a function of the worker's past wage percentile however, it predicts lower probabilities for high past wages as compared to the data. This is because in the model high wage earners are high productivity workers who only transition to lower skill occupations if they suffer a separation or a human capital accumulation shock. Since all the moves down the occupation ladder are driven by a decline in worker productivity, high wage earners are less likely to move down the occupation ladder. The higher incidence of such transitions in the data for high wage earners could be driven by non-productivity related factors such as preference for job flexibility which are not captured by the model. Table 2 shows the values of the estimated parameters. The calibrated values of the production function cannot be compared with any previous estimate. These values along with the human capital production functions and the job offer arrival probabilities of the employed workers determine the training decisions of the firms and the workers in each occupation group. Under the current calibration, as shown in Figure 13(a) firms in the most productive occupations invest the most in worker training. The production function parameters along with the job finding probability in each occupation also play an important role in the search strategies of workers across the age and productivity dimension. This interplay between the two can be seen in Figure 13(b). Young workers with low levels of human capital search in lower productivity occupations where the jobs are easier to find and the prospective matches are feasible. As their productivity evolves over the course of their careers, they start searching for higher productivity jobs. However, after a certain age threshold all workers search in the low productivity occupation because the jobs are easier to find. This is because at older ages the difference in the value a worker gets from a job in each occupation shrinks and the job search decision is driven by the differences in job finding rates which are constant across age.

The value of the bargaining power parameter is within the range of values estimated in the literature with on-the-job search (e.g see Papageorgiou (2013)). The human capital transition function parameters, A_k, α and σ_k are estimated from the average wage growth profile over the life cycle (Figure 12) as well as the transitions of workers towards lower productivity jobs as they become older (Table 1-column 1). The value of the on-the-job search parameter, λ_0, governs the transitions of workers across occupation groups without an intervening unemployment spell. The exogenous job destruction parameter is calibrated to achieve a reasonable steady state rate of unemployment. Under the current calibration, the steady state rate of unemployment is 4.58%. Notice that in this model, the unemployment rate is not only affected by the exogenous job destruction rate but also the search strategies of the workers. If all workers search in occupation 3 with the lowest job finding rate then the steady state unemployment rate would be higher for any given value of δ. Finally, the means of the education specific distribution from which workers draw their initial productivity, helps match the proportion of workers from each educational group working in different occupation categories.

4.3 Importance of Skill Depreciation

There are two forces which push older workers towards low productivity jobs, high job finding rates in low skilled occupations and skill depreciation which leads to less output being produced in high skill occupations. The importance of skill depreciation for matching the empirical facts can seen from table 3. Here I perform a counter-factual experiment in which I set $\sigma_k = 0$ for all k, without changing the job finding rates for each occupation sub-market. Thus workers on average only gain skills and their skills do not depreciate with age. For this counter-factual economy I compute the steady state and compare the results to the baseline model with parameter values given in table 2.

As the results show, without skill depreciation workers move towards occupation groups 2 and 3 as they become older even though it is easier to find jobs in lower productivity occupation group 1. About 75% of the workers without a college degree end up working in college occupations while in the baseline model this fraction is about 20%. Similarly, 85% of workers with a bachelor's degree are now working in occupation group 3 whereas the corresponding number in the baseline model is 7%. Hence, not surprisingly, the model does not produce the U-shape of over-education and instead the share of workers with college degrees working in non-college occupations declines with age. This exercise shows that human capital skill depreciation parameters play an important role in matching the empirical facts.

5 Model Applications

Having solved for the steady state of the model and matched the salient features of the data, one can recover the vacancy posting costs in each occupation using the free entry condition which stipulates that ex-ante profits of all firms in each occupation sub-market are 0. The model can then used for counter-factual analysis. The equilibrium nature of the model, with a substantial role for the firm in the career outcomes of workers, means that the model can be used to evaluate various policies and hypotheses and to simulate the effects of long run structural changes in the labor market on the careers of workers. In this section I describe the vacancy posting decisions of firms to back out the vacancy posting costs and then I analyze two types of structural change within the framework my model which lead to "job-polarization" and discuss the consequences on the careers of workers.

5.1 Vacancy Posting Decisions

Using the decisions of the workers, the steady state distribution and a specification of the matching function, one can back out the vacancy posting costs, which are assumed to be occupation specific, and these are used to conduct counterfactual experiments in section 5. I follow Lise and Robin (2014) for characterizing the vacancy posting decisions of the firms to back out these costs. Denote by $u_k(x)$ as the measure of workers who are unemployed of type x and searching in occupation submarket k. Similarly, denote by $e_k(x)$ as the measure of workers of type x and working in occupation k. For each occupation sub-market k, the effective search effort is

$$l_k = \int u_k(x)\,dx + \lambda_0 \sum_{i \neq k} f_{k,i}(x) \int e_i(x)\,dx \tag{14}$$

where λ_0 is the search effort of the employed workers relative to the unemployed and effective search effort of the employed workers in occupation k consists of all workers in ocupations $i \neq k$ such that they have a higher surplus in submarket k, which implies $f_{k,i}(x) = 1$. This is because only workers who have a higher surplus in occupation k will accept a job offer from that sub-market if they are already employed in occupation i and receive an offer on the job.

Denote by v_k as the number of vacancies posted by firms in sub-market k. The total measure of meetings in occupation k, m_k, is given by a Cobb-Douglas matching function

$$m_k \equiv min\left\{\zeta l_k^{\nu} v_k^{1-\nu}, l_k, v_k\right\}$$

The job finding probability for workers in occupation k, λ_k, can be written as $\lambda_k = m_k/l_k$. Similarly, $q_k = m_k/v_k$ is the probability per vacancy in sub-market k that a firm meets a searching worker. Given that the matching function is assumed to be Cobb-Douglas, the probability q_k can be written as a function of market tightness $\theta_k = v_k/l_k$.

The value of posting a vacancy can now be written as:

$$V_k = -c_k + q(\theta_k)\left[\int J_k(x,u)\frac{u_k(x)}{l_k}\,dx + \lambda_0 \sum_{\{i \neq k\}} f_{k,i}(x) \int J_k(x,i)\frac{e_i(x)}{l_k}\,dx\right] \tag{15}$$

Equilibrium free entry condition would imply that $V_k = 0$ which can be used to back out vacancy posting costs c_k.

5.2 Job Polarization

Consider a change in the relative productivity of one occupation submarket with respect to the others. If that occupation becomes more productive, then workers would try to work in that occupation and this could have significant impact on the careers of the workers along the transition path and in the new steady state.

The empirical work of Acemoglu and Autor (2011) and numerous others has documented that the U.S labor market has gone through a period of polarization in the last three decades whereby middle skills jobs have disappeared while high and low skill jobs have increased. Although middle skill jobs in my setup are predominantly taken up by bachelor degree holders, I can simulate a similar change from the baseline model by decreasing the productivity of occupation group 2, p_2, from its calibrated value. In the new steady state of the model, see Figure 14(a), occupation group 2 has 30% less employment while occupation group 1 and 3 gain employment, with most of the increase going to group 3.

The effects on the overall welfare in the economy can be evaluated over the long run at the new steady state or during the transition to the new steady state. Here I compute the welfare effects in the new steady state and compare outcomes of workers of similar ages in the two economies. Figure 14(b) computes the ratio of average wages in the new economy to baseline model. As can be seen, the ratio is above 1 for all age groups which means that the workers are better off in the economy with lower productivity for middle skill occupation group. The ratio keeps increasing with age as well, this is because more older workers are working in occupation group 3 than before and hence earning higher wages.

To understand the result in Figure 14(b) it is useful to think about the consequences of a decline in occupation productivity on the investment decisions of the workers and the firms. In this new counter-factual economy workers and firms invest in more skills in the middle skill occupation group since the opportunity cost of training goes down[9]. This allows more workers to climb up the occupation ladder and work in the high skill occupation group and thus earn higher wages. Hence the wage growth at the latter part of the career is higher when workers are producing higher output as they are working in the high skill occupation with higher human capital.

5.3 Higher Skill Requirements in Jobs

In the above scenario, workers of all ages are better off in the new steady state of the model. In this sub-section I consider a counter-factual where young workers are better off in the new steady state and the older workers are worse off. The counter-factual scenario I consider here is one where the skill requirements of high productivity jobs increase, creating mismatch between the current skills of the workers and the requirements of the jobs. There has been a lot of debate in the policy and academic circles whether such a structural change in the economy or a "skill-gap" is contributing to the slow recovery in the labor market following the great recession. Some evidence exists that such a change occurs in the aftermath of recessions Hershbein and Kahn (2016) and that it contributes to the phenomenon of jobless recoveries Jaimovich and Siu (2012). Restrepo (2015) builds a model featuring such a structural shift that leads to a jobless recovery. For a detailed discussion of the "skills-gap" hypothesis see

[9]The rental rate on human capital is now lower in the middle skill occupations

Cappelli (2015).

Some observers have pointed out that if indeed a skills-gap exists in the labor market then firms should hire workers with less skills and provide them with training on the job. The current model features such a mechanism. Once again I will not analyze the transition path to the new steady state but compare the worker outcomes in the long run steady state of a model which features higher skill requirements for high productivity occupations to the baseline model. The results are shown in Figure 15.

With higher skill requirements in occupation group 2 and 3. Workers find it hard to move up the occupation ladder as their path to higher skilled jobs is blocked. This leads to more workers in the lowest skill occupation group (Figure 15(a)). As can be seen from Figure 15(b), young workers earn higher wages in the new economy but older workers earn less. This is because in this counter-factual economy fewer workers at an older age are working in the high skill occupation. Since younger workers are unable to move up the occupation ladder, they invest less time in training and thus earn higher wages. Overall, the net effect on worker welfare is negative under this scenario. It is worthwhile to note that while the two counter-factual exercises produce a similar shift in relative employment, the welfare conclusions are very different.

5.4 Related Literature

As mentioned before, Forsythe (2016) also documents downward occupation mobility while focusing on within firm reallocation. She also documents contemporaneous and long-lasting earnings losses associated with moves to lower quality occupations. While I do not focus exclusively on reallocations within firms, I also find similar patterns of mobility towards lower quality jobs with age.

In a similar vein, Rutledge, Sass, and Ramos-Mercado (2015) find that the set of employment opportunities for workers declines with age and they are more likely to transition to lower quality jobs upon a job switch. They define the quality of a job by the median wages within an occupation group and using the O*NET database find that older workers are less likely to be hired in jobs that require active learning and numerical ability.[10] In another paper with similar results, Belbase et al. (2015) show that workers switch to less cognitively demanding jobs as they age and this is correlated with age-related cognitive decline among individuals. Both these studies focus on workers aged 50 and older while I find that transitions to lower quality jobs is a phenomenon that is present even among prime aged workers.

[10] A recent New York Times article featuring this paper referred to these jobs as "old-persons" jobs.

This paper also relates to the literature on over-education that was started by Freeman (1976). He claimed that there was an excess supply of college graduates in the U.S. labor market in the 1970s because of the declining college wage premium. While the hypothesis of Freeman (1976) was rejected by later researchers, the question of over-education was nevertheless brought to the attention of social scientists and policy makers. A large body of research has tackled the question of over-education at the individual and the aggregate level since then.[11] This literature has documented that, at the individual level, over-education is highly persistent and is associated with lower current as well as future wages. My findings on over-education over the life cycle provide a new fact for this literature as the focus of the earlier studies has been on younger workers.

More recently, Clark, Joubert, and Maurel (2014) show how over-education evolves over the early part of the career and explain why it is so persistent for some individuals. Abel and Deitz (2015) use similar measures of over-education[12] derived from the O*NET data to analyze how the aggregate measure of over-education behaves over the course of the business cycle. My paper is a close complement to their work in terms of defining over-education as a state of the labor market. However, they do not document the life cycle patterns reported in this paper because they restrict their analysis to the early years of a worker's career.

The paper is also related to the literature on worker transition across jobs. Since the seminal work of Jovanovic (1979a) economists have known that workers move to better job matches over time. The more time they spend in the labor market, the more precisely they know about their match quality. This simple model can explain some well known empirical facts such as rising wages with experience (and tenure in a job) and declining job mobility with age. Adding search frictions to such an environment can hamper the learning process and workers take a longer time to move to better job matches (see Papageorgiou (2013) for such a combination). One can also include human capital accumulation and job switching costs to add more persistence to this phenomenon (see Wee (2013) for such an example). Nevertheless, the underlying pattern generated by all such models is that workers should move to better job opportunities with experience (or age).

Finally this paper is also related to the literature that uses search models to rationalize large and persistent earnings losses at displacement. As shown by Davis and von Wachter (2011), the basic DMP model is not able to capture the earnings losses associated with displacement in the data. The model presented in this paper can potentially produce this phenomenon through two channels, job quality and loss of human capital from job displacement. Fully exploring the capabilities of the model to explore the forces behind earnings

[11]See Leuven and Oosterbeek (2011) for an excellent summary of this literature.
[12]They refer to over-education as underemployment.

losses after displacement as done by Jarosch (2014) is beyond the scope of the current paper.

6 Conclusion

In this paper, I document new stylized facts regarding occupation choice over the life cycle and the consequences for wages. I find that workers tend to move towards lower productivity occupations in the middle of their careers and earn lower wages upon such transitions. To explain these facts, I build a life cycle occupational search model with skill accumulation and depreciation. The model features heterogeneous workers and occupations which can be ranked in terms of their productivity. Workers choose occupations to maximize their lifetime earnings and also invest in human capital accumulation. However, unlike the previous literature on human capital accumulation, investment decisions are made jointly by the workers and the firms and not by the worker alone.

As the workers gain skills they are able to climb up the occupation ladder and this explains the declining half of the U-shape of over-education. After reaching a certain age, investments in skill accumulation decline and workers start losing their productivity as depreciation sets in. This leads to a movement down the occupation ladder and the proportion of over-educated workers rises with age. The model does a good job of matching the empirical facts and I show that skill depreciation is the key mechanism for matching the set of documented empirical facts.

The model can be used to determine the effects of structural change in the labor market on the careers of workers. In particular, I use the model to simulate polarization in the labor market driven by a decline in the relative productivity of the middle skill occupation group. In another counter-factual experiment, I simulate the impact of an increase in skill requirements of jobs. The results show that while the employment effects of both types of structural change are similar, the welfare consequences are very different and would require different policy prescriptions.

The model can also be used to quantitatively evaluate labor market policies such as unemployment insurance and hiring subsidies for firms. In a typical labor search model, unemployment insurance suppresses the job finding probability of the workers due to higher reservation wages, leading to a higher unemployment rate in equilibrium. However, in a model with heterogeneous workers and firms with complementaries among the two sides of the market, higher unemployment insurance would lead workers to search for high productivity jobs leading to higher life-time earnings. The welfare calculations of higher unemployment insurance in such a model become ambiguous and depend on the parameters of the model.

This point has already been made by Acemoglu and Shimer (1999), albeit in a normative way. In the current paper, unemployment insurance has an additional impact on life cycle earnings of workers through the human capital investment channel. Since human capital investments depend upon the type of jobs a worker get matched with, the generosity of the benefit system could have long term effects on the careers of workers.

The empirical patterns documented in this paper also have important consequences for evaluation of pension policies that affect the retirement decisions of workers. Since workers are unable to hold high productivity jobs due to depreciating skills, policies to extend working age should be complemented with training programs that allow workers to update their skills. However, such an analysis would require the model to be extended to allow for savings and retirement decisions. Such extensions and evaluations of unemployment insurance policies are left for future work.

Ammar Farooq: Georgetown University

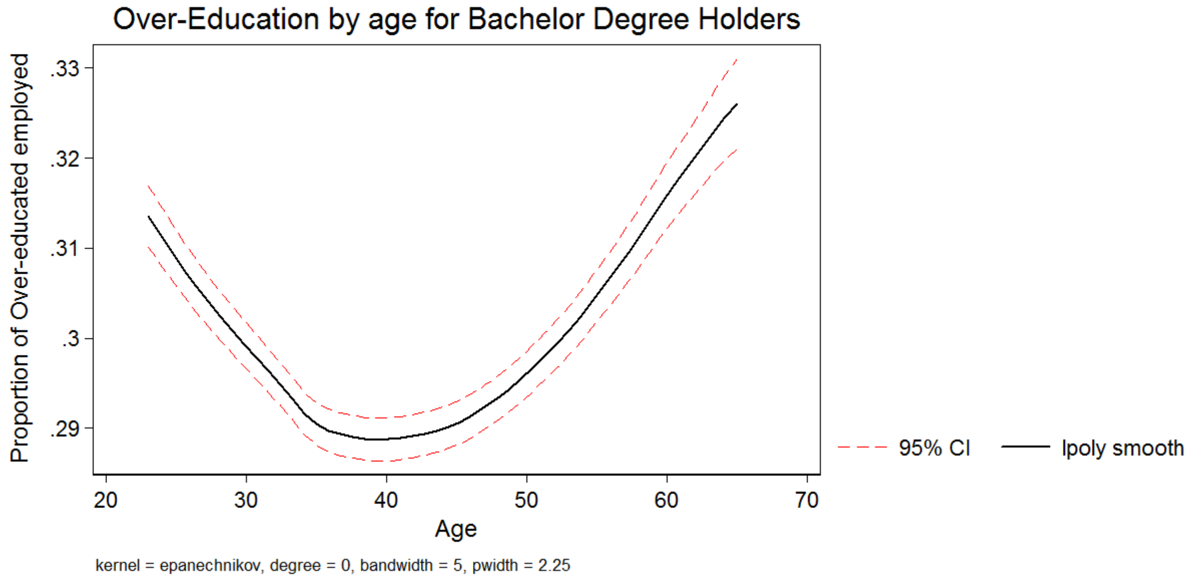


Figure 1: Data from CPS 2003-2010, merged with O*NET data

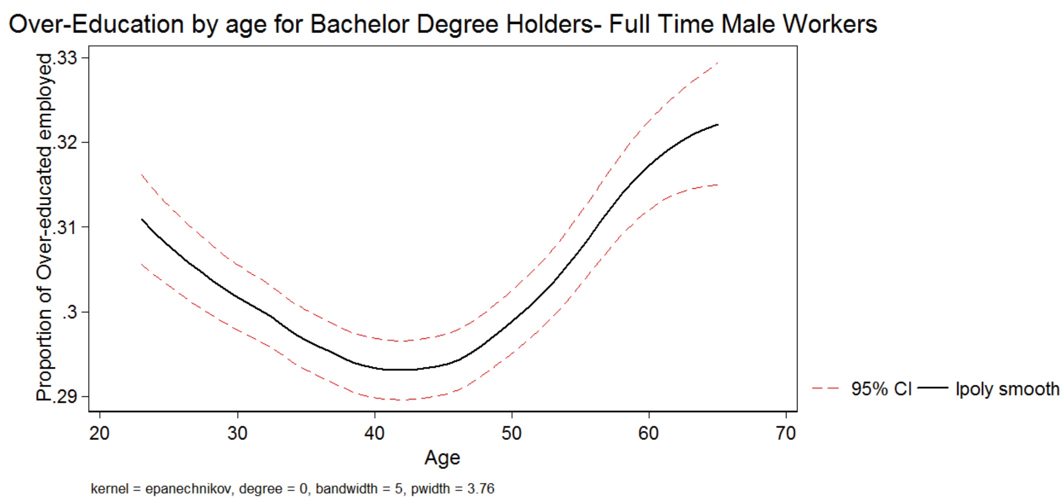


Figure 2: Sample restricted to Male Full Time Workers from CPS

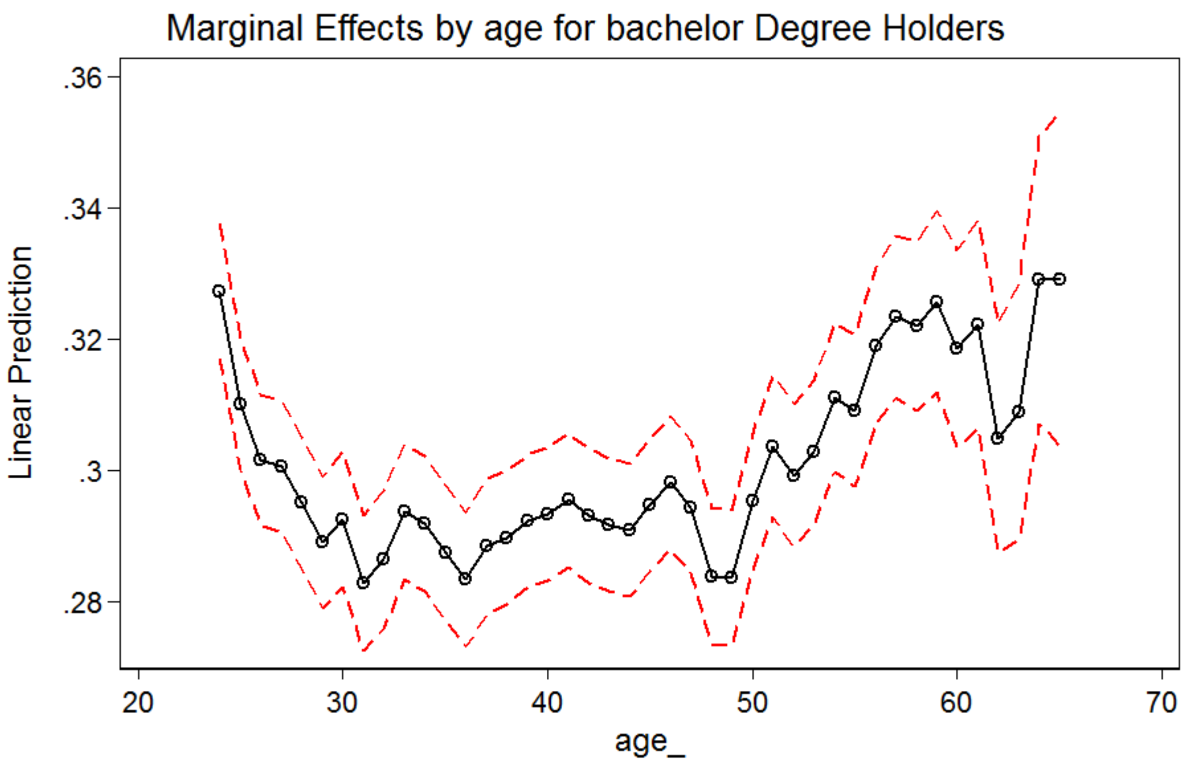
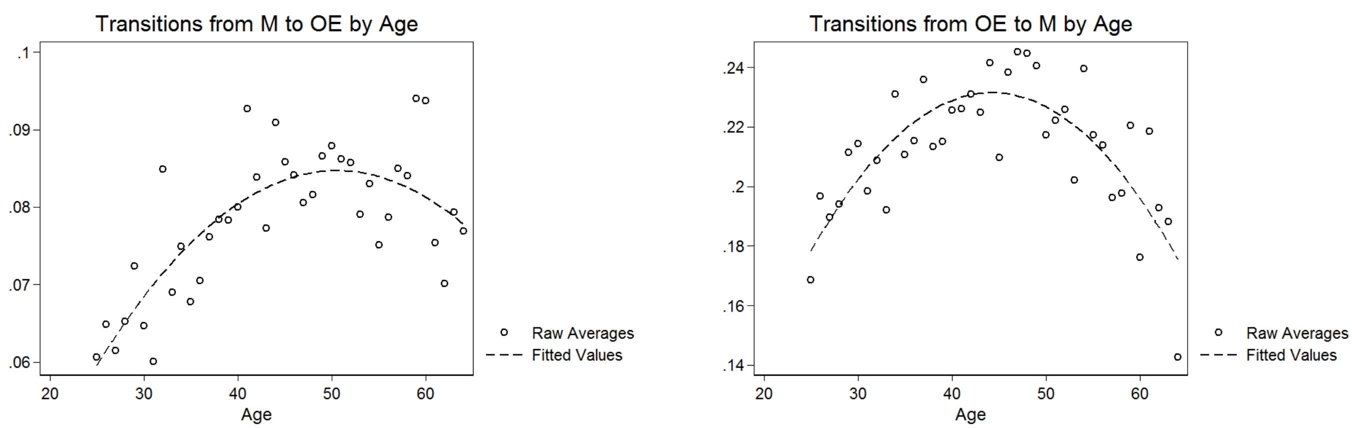


Figure 3: Controlling for Demographic Factors for explaining Over-education



(a) **(b)**

Figure 5: Transitions to Over-education (OE) by Age

Figure 4: Subjective Measure of Over-education from NSCG, 2003-2012

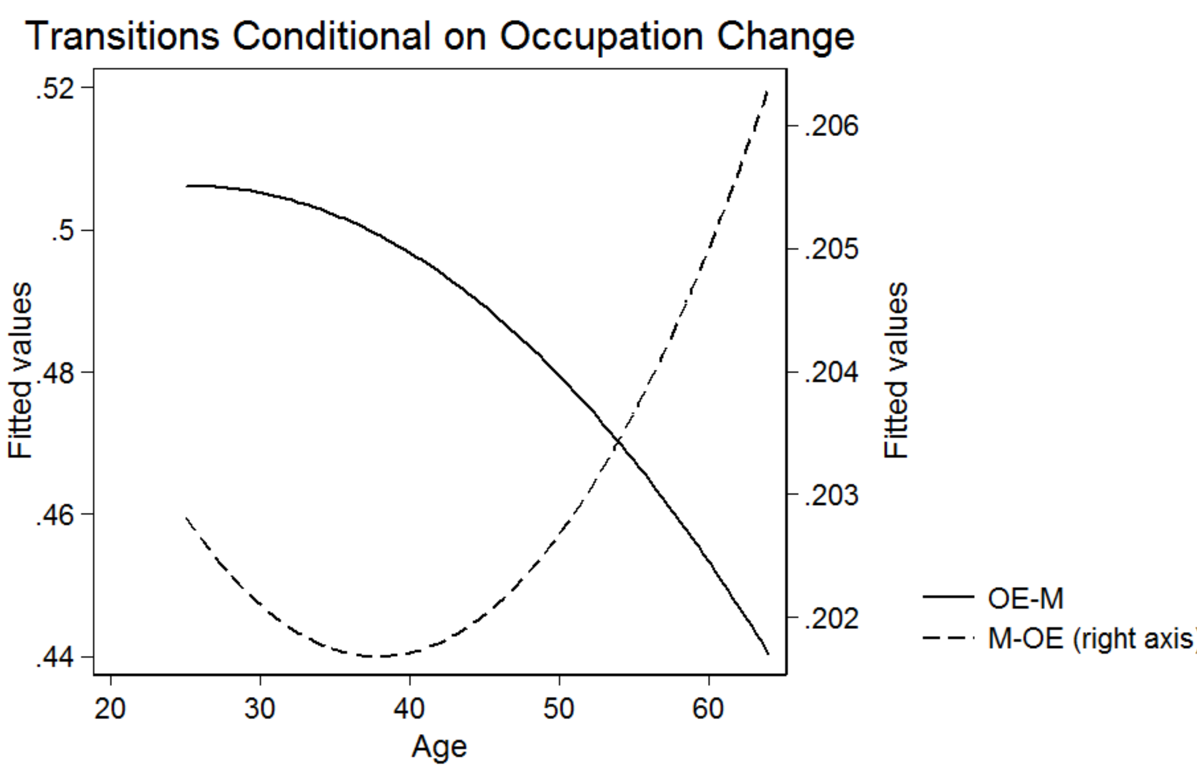


Figure 6: Transitions to Over-education (OE) by Age Conditional on Occupation Change

Figure 7: One year Wage Growth, Computed from CPS-MORG

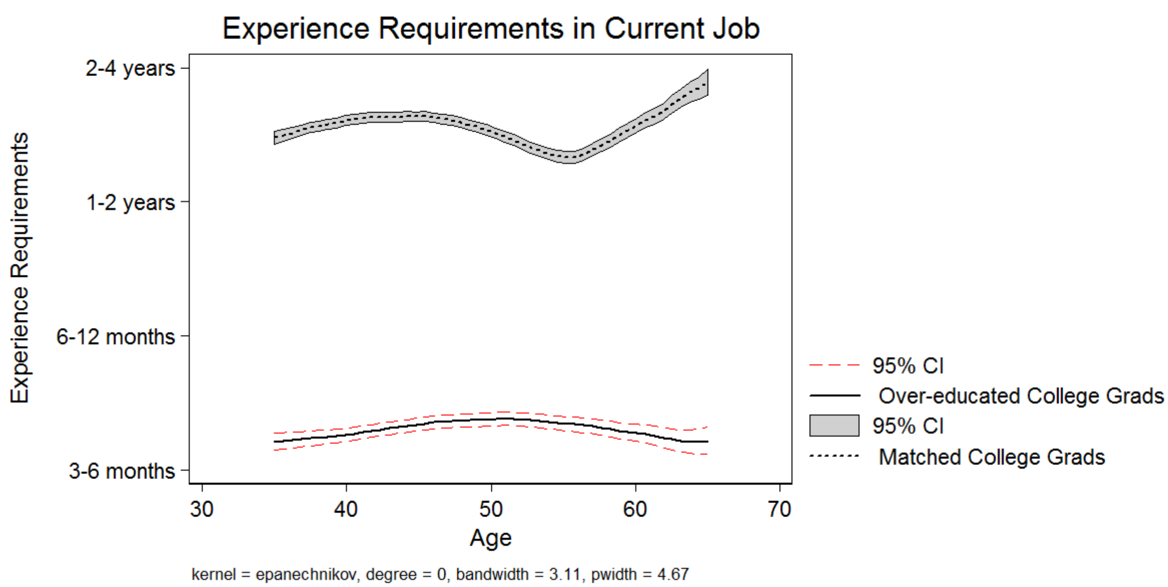


Figure 8: CPS Data merged with experience requirements from O*NET

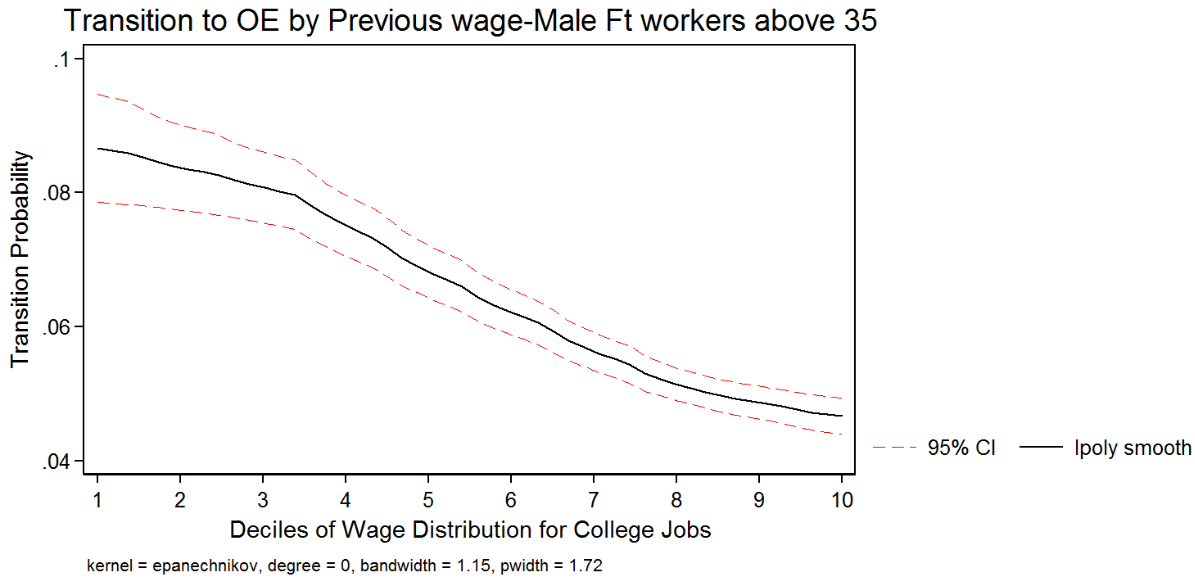


Figure 9: Transition to OE as a function of past occupation wage, CPS MORG

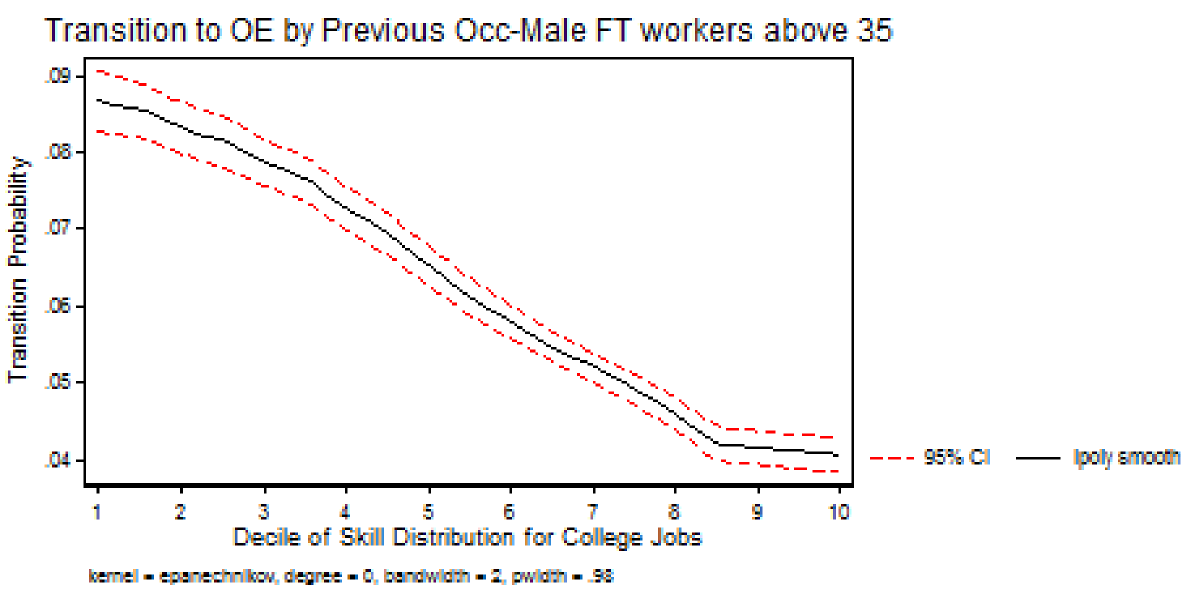


Figure 10: Transition to OE as a function of past occupation cognitive skill index

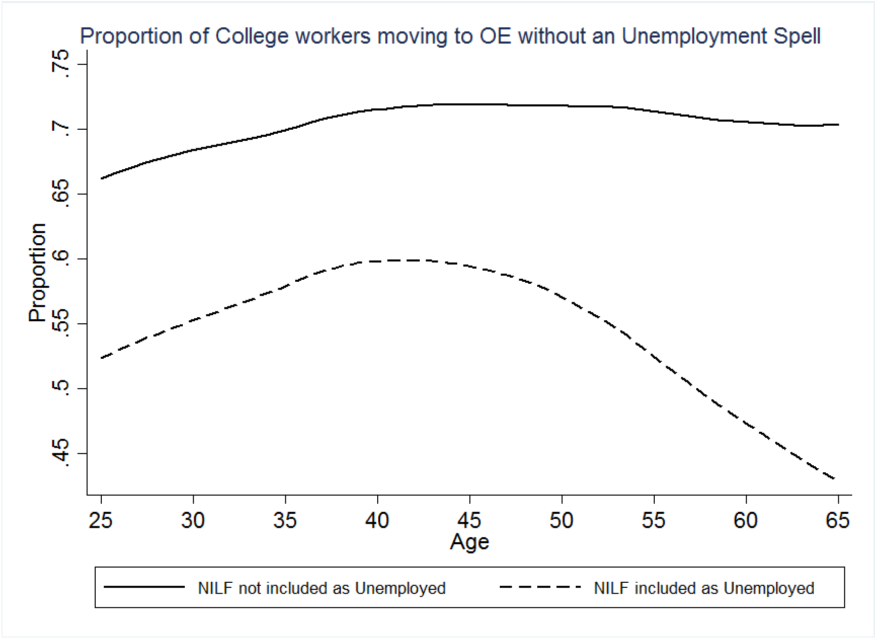
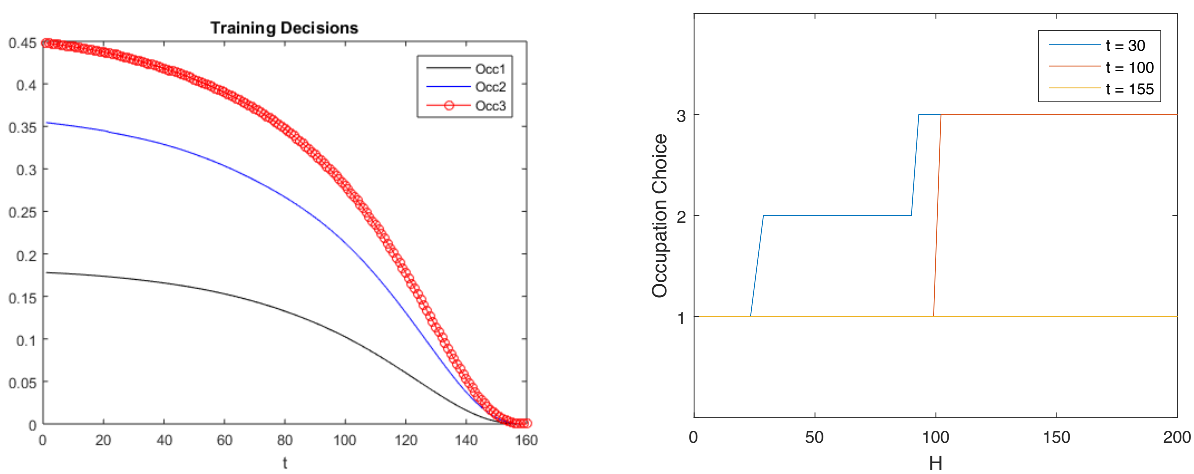


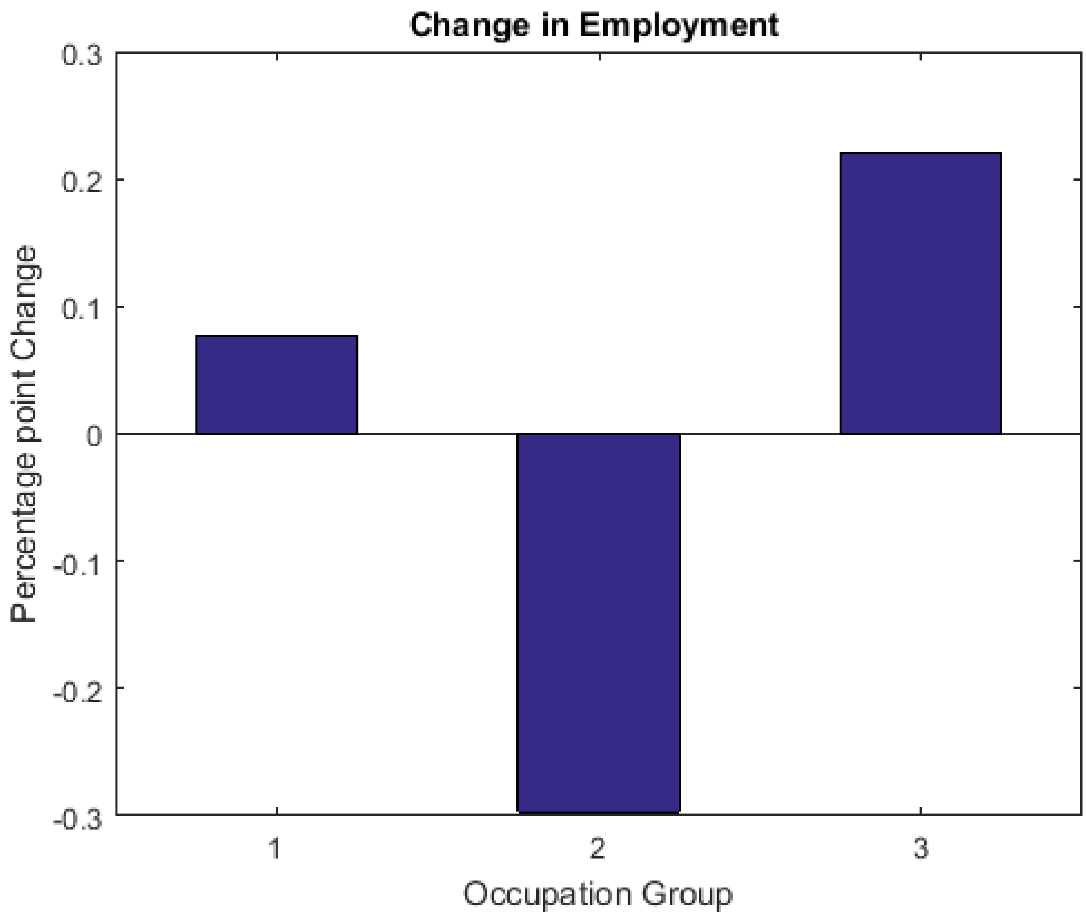
Figure 11: Transition to OE through Employment, CPS

Figure 12: Targeted Moments

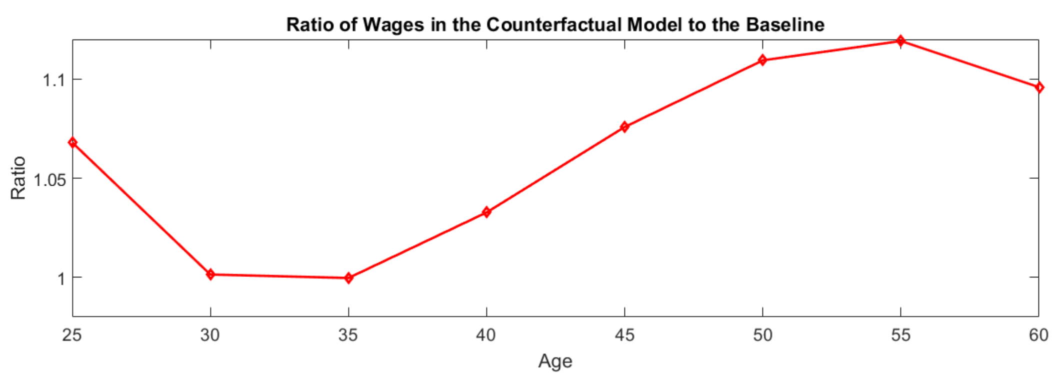


(a) **(b)**

Figure 13: Model Results

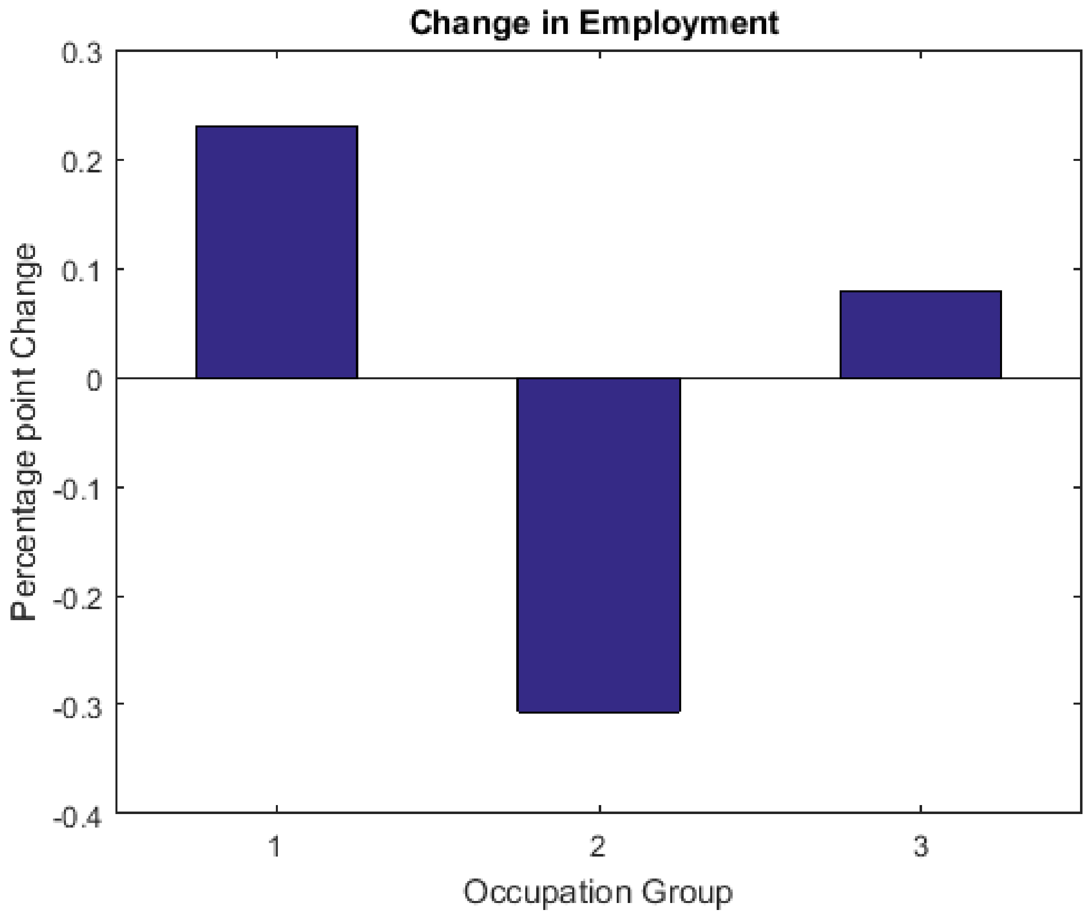


(a) Employment Changes

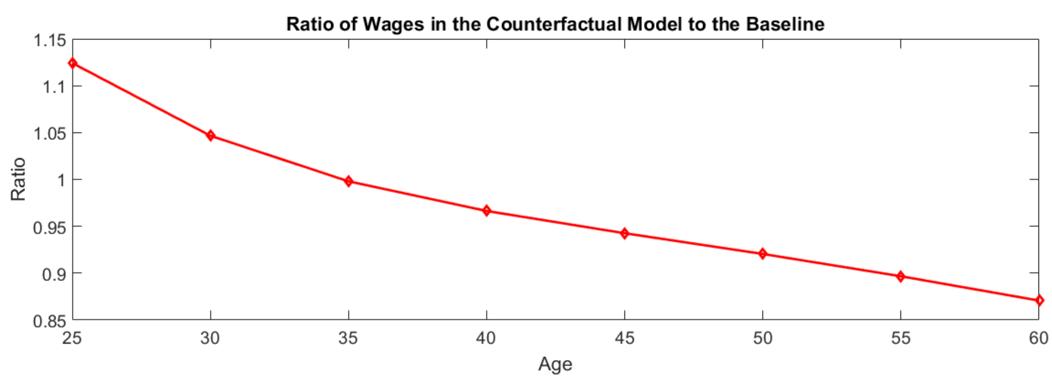


(b) Ratio of Wages

Figure 14: Effect of Higher Skill Requirements on Welfare and Jobs



(a) Employment Changes



(b) Ratio of Wages

Figure 15: Effect of Higher Skill Requirements on Welfare and Jobs

Moment	Model	Target	Moment	Model	Target
% OE in bin 25-29	0.395	0.318	% “Matched” More than College Workers	0.908	0.908
% OE in bin 30-34	0.289	0.299	% “Under-qualified” Non-College Workers	0.197	0.212
% OE in bin 35-39	0.263	0.300	% Bachelor workers in Occ 3	0.086	0.072
% OE in bin 40-44	0.263	0.301	% More than Bachelor workers in Occ 3	0.319	0.387
% OE in bin 45-49	0.274	0.289	Ratio of wages: OE workers to Non-College Workers	1.054	1.073
% OE in bin 50-54	0.303	0.309	Unemployment Rate	0.046	0.050
% OE in bin 55-59	0.327	0.328			
% OE in bin 60-64	0.346	0.334			

Table 1: Model Fit

Parameter	Value	Parameter	Values	Parameter	Values
P_1	3.20	μ_{nc}	1.820	σ_3	0.121
P_2	4.90	μ_b	3.275	μ_z	-0.05
P_3	6.85	μ_{mc}	4.203	$Var(z)$	0.100
τ_1	0	A_1	0.075	b	0.280
τ_2	-50	A_2	0.155		
τ_3	-200	A_3	0.175		
q	0.365	α	0.633		
λ_0	0.155	σ_1	0.088		
δ	0.012	σ_2	0.119		

Table 2: Calibrated Parameters

Moment	No Depreciation	Baseline	Moment	No Depreciation	Baseline
% OE in bin 25-29	0.247	0.395	% ”Matched” More than College Workers	0.989	0.908
% OE in bin 30-34	0.124	0.289	% ”Under-qualified” Non-College Workers	0.753	0.212
% OE in bin 35-39	0.042	0.263	% Bachelor workers in Occ 3	0.653	0.072
% OE in bin 40-44	0.009	0.262	% More than Bachelor workers in Occ 3	0.842	0.387
% OE in bin 45-49	0.002	0.274	Ratio of wages OE workers to Non-College workers	0.643	1.073
% OE in bin 50-54	0.001	0.303	Unemployment Rate	0.059	0.045
% OE in bin 55-59	0.003	0.327			
% OE in bin 60-64	0.013	0.346			

Table 3: Model Without Depreciation of Worker Productivity

References

Abel, Jaison and Richard Deitz. 2015. "Underemployment in the early careers of college graduates following the Great Recession." In *Education, Skills, and Technical Change: Implications for Future US GDP Growth*. University of Chicago Press.

Acemoglu, Daron and David Autor. 2011. "Skills, tasks and technologies: Implications for employment and earnings." *Handbook of labor economics* 4:1043–1171.

Acemoglu, Daron and Robert Shimer. 1999. "Efficient unemployment insurance." *Journal of Political* 8.

Albrecht, James and Susan Vroman. 2002. "A matching model with endogenous skill requirements." *International Economic Review* 43 (1):283–305.

Anderson, Axel and Lones Smith. 2010. "Dynamic matching and evolving reputations." *The Review of Economic Studies* 77 (1):3–29.

Belbase, Anek, Mashfiqur Khan, Alicia H Munnell, and Anthony Webb. 2015. "Slowed or Sidelined? The Effect of'Normal'Cognitive Decline on Job Performance Among the Elderly." *Center for Retirement Research at Boston College Working Paper* (2015-12).

Cahuc, Pierre, Fabien Postel-Vinay, and Jean-Marc Robin. 2006. "Wage bargaining with on-the-job search: Theory and evidence." *Econometrica* 74 (2):323–364.

Cappelli, Peter H. 2015. "Skill Gaps, Skill Shortages, and Skill Mismatches Evidence and Arguments for the United States." *ILR Review* :0019793914564961.

Clark, Brian, Clement Joubert, and Arnaud Maurel. 2014. "The Career Prospects of Overeducated Americans." Working Paper 20167, National Bureau of Economic Research.

Davis, Steven J and Till von Wachter. 2011. "Recessions and the Costs of Job Loss." *Brookings Papers on Economic Activity* :1.

Dey, Matthew S and Christopher J Flinn. 2005. "An equilibrium model of health insurance provision and wage determination." *Econometrica* 73 (2):571–627.

Flinn, Christopher, Ahu Gemici, and Steven Laufer. 2016. "Search, Matching and Training." .

Forsythe, Eliza. 2016. "Downward Occupational Mobility within Firms." *Working paper* .

Freeman, Richard. 1976. "The Overeducated American." .

Gervais, Martin, Nir Jaimovich, Henry E. Siu, and Yaniv Yedid-Levi. 2014. "What Should I Be When I Grow Up? Occupations and Unemployment over the Life Cycle." Working Paper 20628, National Bureau of Economic Research.

Groes, F., P. Kircher, and I. Manovskii. 2014. "The U-Shapes of Occupational Mobility." *The Review of Economic Studies* .

Hershbein, Brad J and Lisa B Kahn. 2016. "Do Recessions Accelerate Routine-Biased Technological Change? Evidence from Vacancy Postings." .

Huggett, Mark, Gustavo Ventura, and Amir Yaron. 2011. "Sources of Lifetime Inequality." *American Economic Review* 101 (7):2923–54.

Jaimovich, Nir and Henry E Siu. 2012. "The trend is the cycle: Job polarization and jobless recoveries." Tech. rep., National Bureau of Economic Research.

Jarosch, Gregor. 2014. "Searching for job security and the consequences of job loss." Tech. rep., Mimeo, University of Chicago, USA.

Jovanovic, Boyan. 1979a. "Firm-specific Capital and Turnover." *The Journal of Political Economy* :1246–1260.

———. 1979b. "Job matching and the theory of turnover." *The Journal of Political Economy* :972–990.

Lentz, Rasmus and Nicolas Roys. 2015. "Training and Search on the Job." Tech. rep., National Bureau of Economic Research.

Leuven, Edwin and Hessel Oosterbeek. 2011. "Overeducation and mismatch in the labor market." *Handbook of the Economics of Education* 4:283–326.

Lise, Jeremy and Jean-Marc Robin. 2014. "The Macro-dynamics of Sorting between Workers and Firms." .

Menzio, Guido, Irina A Telyukova, and Ludo Visschers. 2012. "Directed Search over the Life Cycle." Tech. rep., National Bureau of Economic Research.

Papageorgiou, Theodore. 2013. "Learning your comparative advantages." *The Review of Economic Studies* :rdt048.

Restrepo, Pascual. 2015. "Skill Mismatch and Structural Unemployment." .

Rubinstein, Yona and Yoram Weiss. 2006. "Post schooling wage growth: Investment, search and learning." *Handbook of the Economics of Education* 1:1–67.

Rutledge, Matthew S, Steven A Sass, and Jorge D Ramos-Mercado. 2015. "How Does Occupational Access for Older Workers Differ by Education?" *Available at SSRN 2658195* .

Sanders, Carl and Christopher Taber. 2012. "Life-cycle wage growth and heterogeneous human capital." *Annual Review of Economics* 4:399–425.

Shimer, Robert. 2012. "Reassessing the ins and outs of unemployment." *Review of Economic Dynamics* 15 (2):127–148.

Wee, Shu Lin. 2013. "Born Under a Bad Sign: The Cost of Entering the Job Market During a Recession." .

Appendix A Further Stylized Facts and Robustness Checks

Sample of Workers with more than a Bachelor's Degree

This section shows the incidence of over-education over the life cycle for workers who have more than a Bachelor's degree. For this group the over-education measure takes a value of 1 if they are working in a non-college job, where the college jobs are defined as in section 2. When I use the National Survey of College Graduates, I use the same question as the main text to determine over-educated workers among this group. Figures A1-A2 show that for this education group, the incidence of over-education also rises over the course of the life cycle.

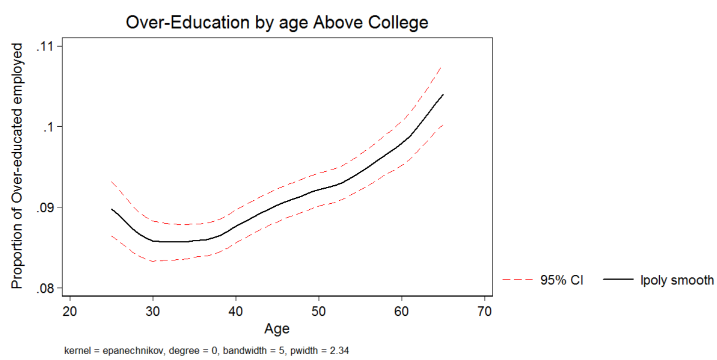


Figure A1: CPS Data- Sample of Workers with More than a Bachelor's degree

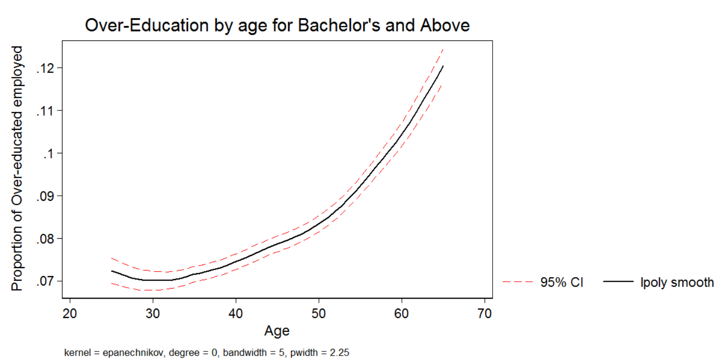


Figure A2: NSCG Data- Sample of Workers with More than a Bachelor's degree

Destination Occupation Characteristics

This section documents the wage percentiles and cognitive skill percentiles in the destination occupation of workers who make a downward transition. The cognitive skill percentiles are

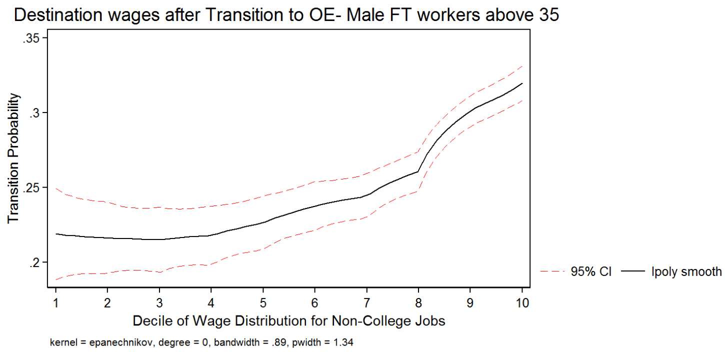


Figure A3: Wages declies of Workers who make Downward Transitions

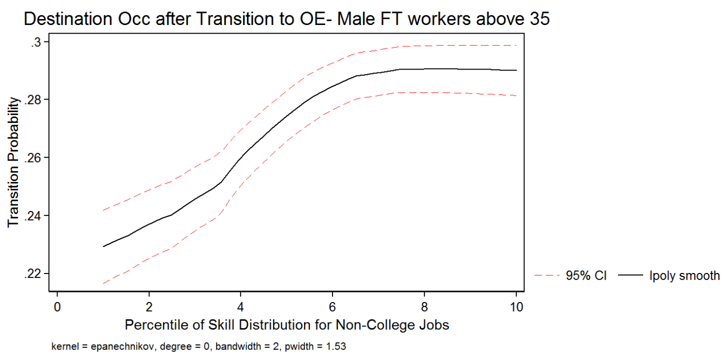


Figure A4: Cognitive Skill deciles of Workers who make Downward Transitions

based on the cognitive skill score for each occupation from Acemoglu and Autor (2011). For both these measures, the percentiles are computed by restricting the sample to workers who are working in non-college jobs. Both these measures are used as proxies for worker productivity to infer the relative productivity of over-educated workers relative to non-college workers. Figures A3 and A4 show that whether one defines productivity through wages or cognitive skill requirements, workers who make downward transitions end up near the top of these distributions among non-college workers. This empirical fact is consistent with the model.

Wage Profiles of Over-Educated Workers

Figure A5 shows the wage premium in cross-sectional data CPS. The wage premium for over-educated college graduates is lower than matched workers, specially at younger and older ages. Another striking feature of this figure is that the overall college wage premium that is considered the main benefit from going to college is entirely driven by adequately matched college graduates while the 30-40 percent of over-educated college graduates receive relatively less premium on their investment.

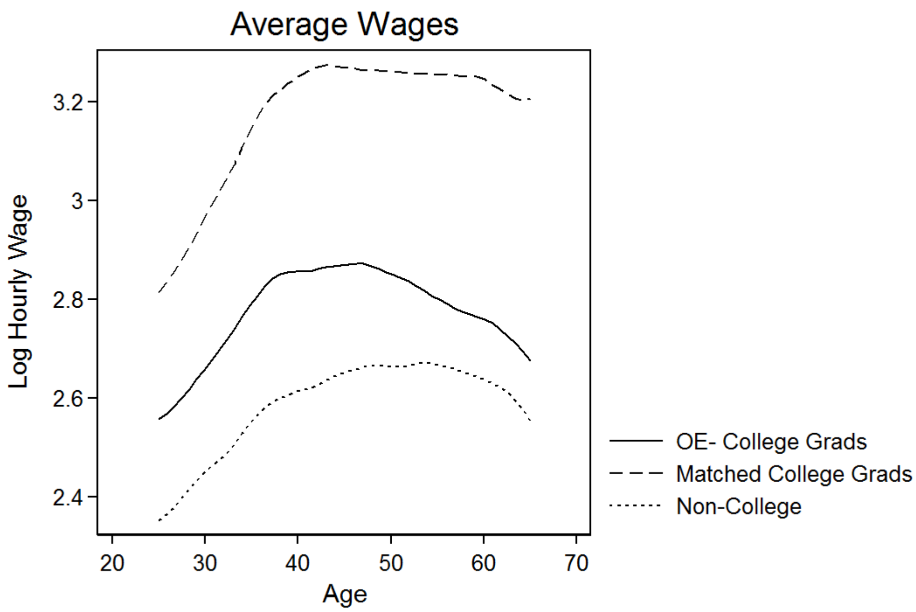
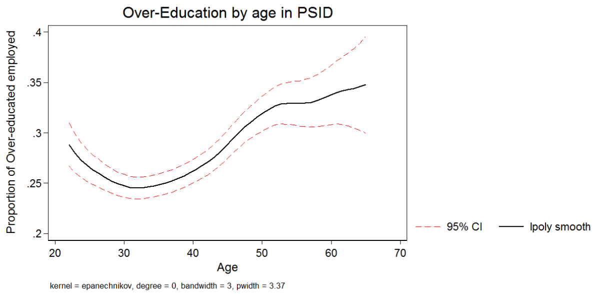


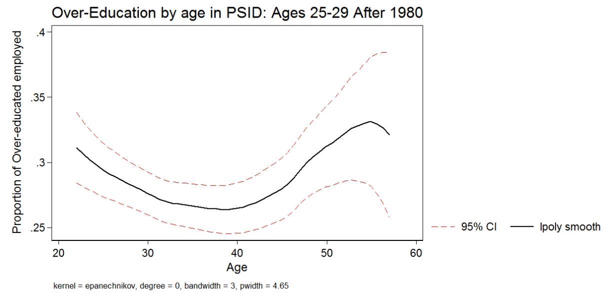
Figure A5: Wage Premium over Age across Different Groups

The U-shape of Over-Education from PSID

For the PSID sample, I restrict attention to male head of households with a bachelor's degree. Figure A6 shows the results from the analysis of PSID data for different cohorts. The U-shape is more pronounced for older cohorts that it is for younger ones. For this analysis I used the occupational crosswalks to identify high skilled occupations in 1980s and 1970s. Misclassification of jobs to high skilled categories in the 1970s might explain the rise in U-shape for earlier cohorts in the PSID. It is worth noting that the proportion of workers with a bachelor's degree in the PSID who are over-educated in their jobs is very similar to the proportion documented from CPS-MORG data from 2003-2010.



(a) All Cohorts-BA



(b) Cohorts 25 or younger in 1980 -BA

Figure A6: Over-education in the PSID sample-Bachelor's Degree Holders

Over-education measure and job quality

In this subsection I compare the over-education measure with other measures of occupation quality used in the literature such as occupation median wages and skill requirements of occupations developed by Acemoglu and Autor (2011). Figure A7-A8 show that it is highly correlated with other measures of job quality. In particular, over-educated college workers are working in jobs that have similar characteristics to jobs performed by non-college workers.

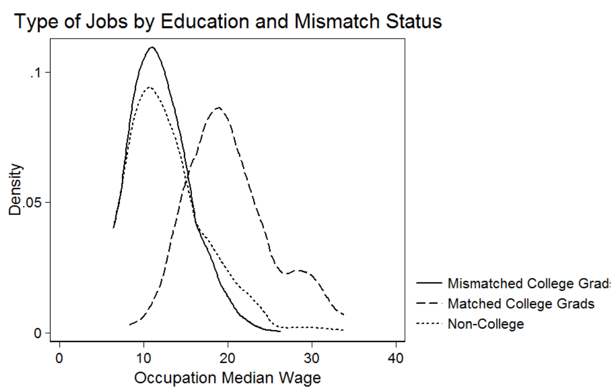


Figure A7: Occupation Median Wage for Jobs done by various groups

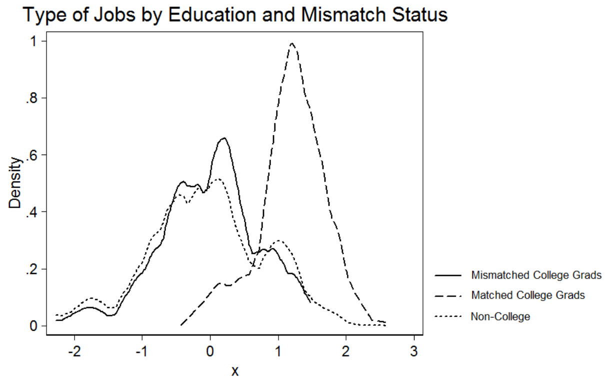
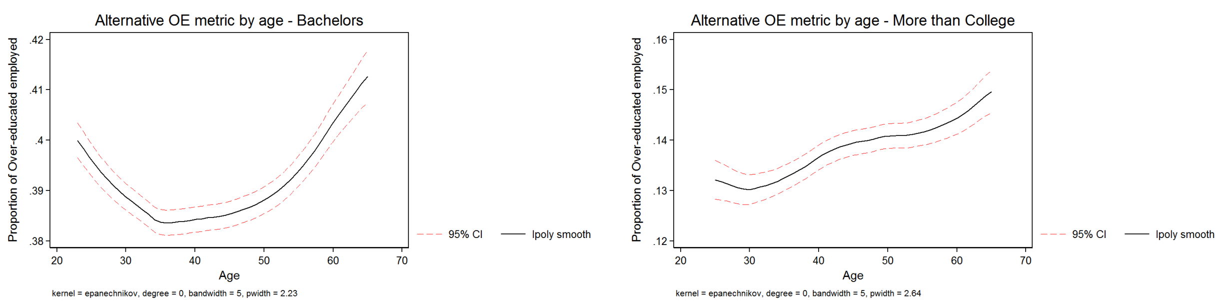


Figure A8: Cognitive Skills for Jobs done by various groups

Alternative Over-Education Measures

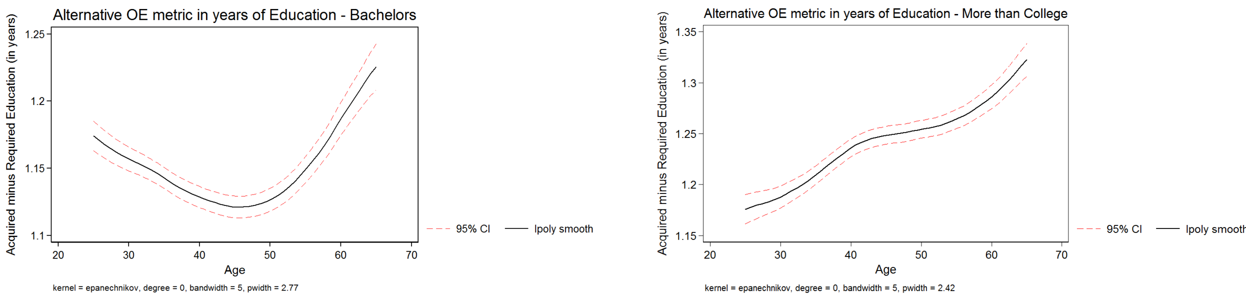
In this section I consider two alternative over-education measures. The first is a measure developed by Abel and Deitz (2015) using the O*NET database in which they consider college jobs as occupations in which at least 50% of the respondents say that at least a bachelor's degree was necessary to perform the job. Using this measure I can construct measures of over-educated college graduates as in section 2. Figure A9 shows the resulting life cycle profiles which are qualitatively similar to figures 1 and 2.



(a) Bachelor's Degree Holders　　**(b)** More than a Bachelor's Degree

Figure A9: Over-education using the metric of Abel and Deitz (2015)

I also develop another over-education measure in terms of years of education required to perform a job. Using the complete distribution of responses in the O*NET database and assigning years of education to each response category, one can calculate the average number of years of education required to work in an occupation. For example, if 75% of the respondents within an occupation agree that a Bachelor's degree is required to perform an occupation and the remaining 25% respond that a Doctoral degree is required, then the average years of education required for that occupation would be $0.75 \times 16 + 0.25 \times 18 = 16.5$. Here the underlying assumption is that a Bachelor's degree is equal to 16 years of education and a Doctoral degree is equal to 18 years of education. This information can then be merged with the worker level datasets as in section 2 to compute a measure of over-education in terms of years of education. The life cycle profile of this measure is shown in figure A10. It also shows that the proportion of college workers who are over-educated in their jobs increases over the life cycle.



(a) Bachelor's Degree Holders **(b)** More than a Bachelor's Degree

Figure A10: Over-education in terms of years of education

Appendix B Computation Details

For the quantitative exercise, I solve the surplus function (12) and the value of unemployment (13) together on a grid for h and three types of occupations using backward induction with $T = 160$. The continuation value at $T = 160$ is assumed to be equal to 0. The investment decision, s_k, is solved by maximizing (12) such that the surplus function and the match output is positive for all values of $s_k \in [0, 1]$. Wages are computed by solving equation (9) and (6) together. For computing the stationary distribution, I simulate $10,000$ agents for 160 time periods which equals 40 years of working life. For the simulation exercise, all the required information is contained in the surplus function, the value of unemployment, the investment function and the wage function.